Primary Mathematics Scheme of Work: Class 4
Year 4 and Year 5

	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6	Week 7
Autumn 1	Number: Place Value				Number: Addition and Subtraction		
Autumn 2	Number: Multiplication and Division			Measurement Y4- Length, Perimeter and Area Y5- Perimeter and Area		Number: Fractions	
Spring 1	Number: Multiplication and Division			Number: Fractions			
Spring 2	Number: Place Value Y4- Decimals Y5- Percentages		Geometry: Shape		Statistics		
Summer 1	Number: Decimals		Measurement: Money		Measurement: Time		
Summer 2	Number: Y4- Consolidation Y5-Negative Numbers		Geometry: Position and Direction		Measurement: Y4-Length, Perimeter and Area Y5-Converting units of measure		Measurement: Y4- Mass and capacity Y5-Volume

Year 4

Place Value

Key concepts (National Curriculum statements)

- count in multiples of 6, 7, 9, 25 and 1000
- find 1000 more or less than a given number
- count backwards through zero to include negative numbers
- recognise the place value of each digit in a four-digit number (thousands, hundreds, tens, and ones)
- order and compare numbers beyond 1000
- identify, represent and estimate numbers using different representations
- round any number to the nearest 10, 100 or 1000
- solve number and practical problems that involve all of the above and with increasingly large positive numbers
- read Roman numerals to 100 (I to C) and know that over time, the numeral system changed to include the concept of zero and place value.

Notes and guidance (non-statutory)

- Using a variety of representations, including measures, pupils become fluent in the order and place value of numbers beyond 1,000, including counting in 10s and 100s, and maintaining fluency in other multiples through varied and frequent practice.
- They begin to extend their knowledge of the number system to include the decimal numbers and fractions that they have met so far.
- They connect estimation and rounding numbers to the use of measuring instruments.
- Roman numerals should be put in their historical context so pupils understand that there have been different ways to write whole numbers and that the important concepts of 0 and place value were introduced over a period of time.

Possible Themes	Key learning points
• Work with numbers less than 10 000 • Understand and use Roman numerals • Explore the history of our number system • Explore ways of representing numbers • Develop skills of estimation	• Order numbers up to and including those with four digits • Write numbers up to and including those with four digits • Read numbers up to and including those with four digits • Read Roman numerals up to C • Understand the difference between the Roman numeral system and the decimal number system • Interpret numbers up to 10 000 on a number line • Represent numbers up to 10 000 using a number line • Use and interpret scales representing measurements with numbers up to 10 000 • Approximate any number by rounding to the nearest 10, 100 or 1000 • Approximate any number with one decimal place by rounding to the nearest whole number • Understand checking as the process of working backwards from the answer to ensure that it makes sense • Understand estimating as the process of finding a rough value of an answer or calculation • Order numbers up to and including those with four digits • Write numbers up to and including those with four digits • Read numbers up to and including those with four digits • Read Roman numerals up to C • Understand the difference between the Roman numeral system and the decimal number system • Interpret numbers up to 10 000 on a number line • Represent numbers up to 10 000 using a number line • Use and interpret scales representing measurements with numbers up to 10 000
Prerequisite	NCETM – Ready to Progress

- ✓ Understand place value in numbers up to three digits
- ✓ Know the Roman numerals I, V and X
- ✓ Read Roman numerals up to XII
- ✓ Use zero as a place holder in two- and three-digit numbers
- ✓ Use and interpret a number line to represent numbers
- ✓ Order and compare numbers up to 10 000
- ✓ Order and compare numbers with one decimal place
- ✓ Know that addition and subtraction are inverses of each other
- ✓ Know that multiplication and division are inverses of each other

Recognise the place value of each digit in a four-digit number (thousands, hundreds, tens, and ones)

Children should be able to:

- Give the value of a digit in a given number e.g. the 7 in 3 274
- Write in figures a given number e.g. four thousand and twenty.
- Recognise a number partitioned like this: 4 000 + 200 + 60 + 3 and be able to read and write the number.
- Create the biggest and smallest whole number with four digits eg. 3, 0, 6, 5
- Find missing numbers in a number sentence e.g. _ +_ = 1249

Order and compare numbers beyond 1000

Children should be able to:

- Find numbers that could go in the boxes to make these correct, □ + □ < 2000, 3000 > □ – □

Identify, represent and estimate numbers using different representations

Children should be able to:

- Answer questions such as, which of these numbers is closest to the answer of 342 – 119: 200 220 230 250 300
- Identify what the digit 7 represents in each of these amounts: £2.70, 7.35m, £0.37, 7.07m

Read Roman numerals to 100 (I to C) and know that over time, the numeral system changed to include the concept of zero and place value

This is new content for the primary national curriculum in England. Suggestions for what children should be able to do include;

- Know what each letter represents in Roman numerals and be able to convert from Roman numeral to our current system (Arabic) and from Arabic to Roman e.g. 76 = _ *in Roman numerals, CLXIX* = _ Arabic numerals.
- Know that the current western numeral system is the modified version of the Hindu numeral system developed in India to include the concept of zero and place value.

identify, represent and estimate numbers using different representations

Children should be able to:

- Answer questions such as, which of these numbers is closest to the answer of 342 – 119: 200 220 230 250 300
- Identify what the digit 7 represents in each of these amounts: £2.70, 7.35m, £0.37, 7.07m

round any number to the nearest 10, 100 or 1000

Children should be able to:

- Explain tips to give someone who is learning how to round numbers to the nearest 10, or 1000.
- Answer questions such as, I rounded a number to the nearest 10. The answer is 340. What number could I have started with? Know what to look for first when you order a set of numbers and know which part of each number to look at to help you.

round decimals with one decimal place to the nearest whole number

- Round these to the nearest whole number. For example:
  9.7, 25.6, 148.3
- Round these lengths to the nearest metre:
  1.5m, 6.7m, 4.1m, 8.9m
- Round these costs to the nearest £:
  £3.27, £12.60, £14.05, £6.50

compare numbers with the same number of decimal places up to two decimal places

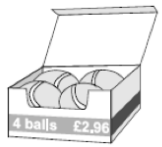
- Place these decimals on a line from 0 to 2:
  0.3, 0.1, 0.9, 0.5, 1.2, 1.9

- Which is lighter: 3.5kg or 5.5kg? 3.72kg or 3.27kg? Which is less: £4.50 or £4.05?
- Put in order, largest/smallest first:
  6.2, 5.7, 4.5, 7.6, 5.2, 99, 1.99, 1.2, 2.1
- Convert pounds to pence and vice versa. For example: Write 578p in £.
- How many pence is £5.98, £5.60, £7.06, £4.00? Write the total of ten £1 coins and seven 1p coins (£10.07)
- Write centimetres in metres. For example, write: 125 cm in metres (1.25 metres)

solve simple measure and money problems involving fractions and decimals to two decimal places.

- These are the prices in a shoe shop. How much more do the boots cost than the trainers? Rosie buys a pair of trainers and a pair of sandals. How much change does she get from £50?

- A box of four balls costs £2.96. How much does each ball cost? Dean and Alex buy 3 boxes of balls between them. Dean pays £4.50. How much must Alex pay? KS2 Paper B level 3



- A full bucket holds 5½ litres. A full jug holds ½ a litre. How many jugs full of water will fill the bucket?
- Harry spent one quarter of his savings on a book. What did the book cost if he saved: £8…£10…£2.40…?
- Gran gave me £8 of my £10 birthday money. What fraction of my birthday money did Gran give me?
- Max jumped **2.25 metres** on his **second** try at the long jump.
  This was **75 centimetres** longer than on his **first** try.

How far in **metres** did he jump on his **first** try?

Reasoning opportunities and probing questions	Mathematical Language	Possible misconceptions
• Show me a four-digit number with a tens unit of '6'. And another. And another … • Kenny writes the number 99 in Roman numerals as IC. Jenny thinks is should be LXXXXVIIII. Do you agree with Kenny or Jenny? • Benny writes the number two thousand and thirty six as '20036'. Do you agree with Benny? • What's the same and what's different: Roman numerals and Base 10 (Denary) number system? • Convince me that 15 rounds to 20 to the nearest 10 • What is the same and what is different: 595, 649, 534 and 634 • Kenny thinks that 4.6 rounds to 4 to the nearest whole number. Do you agree? Explain your answer. • NCETM: Place Value Reasoning	Place value Digit Thousands Hundreds Tens Ones Zero Roman Numeral Estimate Number line Scale	• Some pupils may think that zero is positive as it does not have a negative symbol • Some pupils may use 'base 10 thinking' to write Roman numerals, for example 49 as 'IL' as it is 'one less than fifty' • Some pupils may write four digits numbers literally, for example, four thousand and twenty six as '400026' • Some pupils may ignore place value and simply write the digits mentioned in a number, for example, four thousand and twenty six as '426' • Some pupils may truncate instead of round • Some pupils may misunderstand the rounding process as one that works from the end of the number; for example 347 to the nearest 100 is worked out as 347 à 350 à 400. • Some pupils may round down at the half way point, rather than round up.

Addition and Subtraction

Key concepts (National Curriculum statements)
Pupils should be taught to:

- add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate
- estimate and use inverse operations to check answers to a calculation
- solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why.

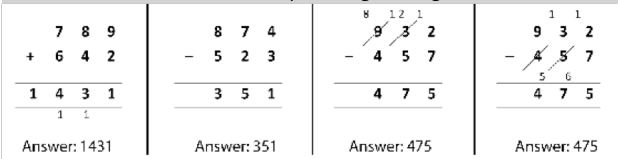
Notes and guidance (non-statutory)

- Pupils continue to practise both mental methods and columnar addition and subtraction with increasingly large numbers to aid fluency

Possible Themes	Key learning points
• Develop mental methods of addition and subtraction • Extend written methods of addition and subtraction • Solve problems involving addition and subtraction	• Find 1000 more than a given number • Find 1000 less than a given number • Use columnar addition for numbers with up to four digits with no carrying required • Use columnar addition for four-digit and two-digit numbers with carrying required • Use columnar addition for four-digit and three-digit numbers with carrying required • Use columnar addition for four-digit numbers with carrying required • Use columnar subtraction for numbers with up to four digits with no exchanging required • Use columnar subtraction for four-digit and two-digit numbers with exchanging required • Use columnar subtraction for four-digit and three-digit numbers with exchanging required • Use columnar subtraction for four-digit and four-digit numbers with exchanging required • Solve two-step problems involving addition and/or subtraction
Prerequisite	NCETM – Ready to Progress

- ✓ Find 100 more or less than a given number
- ✓ Use column addition and subtraction for numbers up to three digits

add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate



estimate and use inverse operations to check answers to a calculation

- Tina has read the first 85 pages in a book that is 150 pages long. Which number sentence could Tina use to find the number of pages she must read to finish the book?

  A 150 + 85 = ☐
  B ☐ – 85 = 150
  C 150 ÷85 = ☐
  D 150 – 85 = ☐

solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why

Children should be able to carry out practical tasks such as that represented here in an Australian classroom.

- Children were asked to individually run the class market stall. They were told they could use mental strategies or the whiteboard provided to assist them in their calculations. The customer (their teacher) would come to purchase some items. Each child was asked to solve a transaction problem involving a single item (calculating change – subtraction) and then a transaction involving two items (adding together values and then calculating change or two subsequent subtractions). They were also asked to explain their thinking and asked how to give the change in a different way (representing money values in various ways).

Children should be able to solve problems such as:

- I have read 134 of the 512 pages of my book. How many more pages must I read to reach the middle?
- There are 8 shelves of books. 6 of the shelves hold 25 books each. 2 of the shelves have 35 books each. How many books altogether are on the shelves?
- I think of a number, subtract 17, and divide by 6. The answer is 20. What was my number?
- You start to read a book on Thursday. On Friday you read 10 more pages than on Thursday. You reach page 60. How many pages did you read on Thursday?

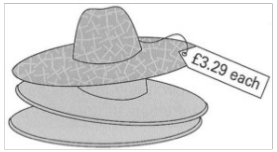
Amir and Lara buy some fruit.

grapes	pineapples	peaches
£2.50 for 1 kg	£1.40 each	£1.99 for a box

Amir buys 2 pineapples and a box of peaches. How much does he pay?

Lara buys half a kilogram of grapes and one pineapple. How much change does she get from £5?

A shop sells sun hats.



Ryan buys some sunglasses for £4.69 and a sun hat. How much change does he get from £10?

Reasoning opportunities and probing questions	Mathematical Language	Possible misconceptions
• Provide examples of column addition and subtraction with hidden digits. Challenge pupils to find these digits and explain their reasoning. • Show me an example of a column addition (that includes carrying) with the answer 2106 • Convince me that 6095 – 3622 = 2473 • NCETM: Addition and Subtraction Reasoning	Addition Subtraction Sum, Total Difference, Minus, Less Column addition Column subtraction Exchange Operation Estimate	• Some pupils incorrectly assume and use commutativity within column subtraction; for example: 4 1 2 6 – 3 7 3 4 1 6 1 2 • Some pupils may not use place value settings correctly (especially when the numbers have a different number of digits)

Multiplication and Division

Key concepts (National Curriculum statements)
Pupils should be taught to:

- recall multiplication and division facts for multiplication tables up to 12 × 12
- use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers
- recognise and use factor pairs and commutativity in mental calculations
- multiply two-digit and three-digit numbers by a one-digit number using formal written layout
- solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects.

Notes and guidance (non-statutory)

- Pupils continue to practise recalling and using multiplication tables and related division facts to aid fluency.
- Pupils practise mental methods and extend this to three-digit numbers to derive facts, (for example 600 ÷ 3 = 200 can be derived from 2 x 3 = 6).
- Pupils write statements about the equality of expressions (for example, use the distributive law 39 × 7 = 30 × 7 + 9 × 7 and associative law (2 × 3) × 4 = 2 × (3 × 4)).
- They combine their knowledge of number facts and rules of arithmetic to solve mental and written calculations for example, 2 x 6 x 5 = 10 x 6 = 60.
- Pupils solve two-step problems in contexts, choosing the appropriate operation, working with increasingly harder numbers. This should include correspondence questions such as the numbers of choices of a meal on a menu, or three cakes shared equally between 10 children

Possible Themes	Key learning points
• Develop mental arithmetic skills • Develop knowledge of multiplication tables • Explore place value • Develop written methods of multiplication • Solve problems involving multiplication and division	• Recall and use multiplication facts for the 6 times table • Recall and use multiplication facts for the 7 times table • Recall and use multiplication facts for the 9 times table • Recall and use multiplication facts for the 11 times table • Recall and use multiplication facts for the 12 times table • Recall and use division facts for the 6 times table • Recall and use division facts for the 7 times table • Recall and use division facts for the 9 times table • Recall and use division facts for the 11 times table • Recall and use division facts for the 12 times table • Use knowledge of factor pairs (commutativity) when multiplying and dividing mentally including multiplying three numbers together • Know the effect of multiplying by 0 and 1 and dividing by 1 • Use the distributive law to multiply a two-digit number by a one-digit number • Use short multiplication to multiply a two-digit number by a one-digit number • Use short multiplication to multiply a three-digit number by a one-digit number • Identify when a scaling or correspondence problem can be solved using multiplication or division
Prerequisite	NCETM – Ready to Progress

✓ Recall multiplication and division facts for 2, 3, 4, 5, 8 and 10 multiplication tables ✓ Understand that multiplication and division are inverse operations	recall multiplication and division facts for multiplication tables up to 12 × 12 Children should be able to: ➢ Pupils continue to practise recalling and using multiplication tables and related division facts to aid fluency. ➢ *e.g. One orange costs nineteen pence. How much will three oranges cost?* ➢ *What is twenty-one multiplied by nine?* ➢ *How many twos are there in four hundred and forty?* use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers Children should be able to: ➢ Pupils practise mental methods and extend this to three-digit numbers to derive facts, for example 200 × 3 = 600 into 600 ÷ 3 = 200. ➢ *e.g. Divide thirty-one point five by ten.* ➢ *Ten times a number is eighty-six. What is the number?* recognise and use factor pairs and commutativity in mental calculations Children should be able to: ➢ Pupils write statements about the equality of expressions (e.g. use the distributive law 39 × 7 = 30 × 7 + 9 × 7 and associative law (2 × 3) × 4 = 2 × (3 × 4)). They combine their knowledge of number facts and rules of arithmetic to solve mental and written calculations e.g. 2 x 6 x 5 = 10 x 6. ➢ *e.g. Understand and use when appropriate the principles (but not the names) of the commutative, associative and distributive laws as they apply to multiplication:* ➢ *Example of commutative law 8 × 15 = 15 × 8* ➢ *Example of associative law 6 × 15 = 6 × (5 × 3) = (6 × 5) × 3 = 30 × 3 = 90* ➢ *Example of distributive law 18 × 5 = (10 + 8) × 5 = (10 × 5) + (8 × 5) = 50 + 40 = 90* solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects Children should be able to: ➢ Pupils solve two-step problems in contexts, choosing the appropriate operation, working with increasingly harder numbers. This should include correspondence questions such as the numbers of choices of a meal on a menu, or three cakes shared equally between 10 children. ➢ *e.g. 185 people go to the school concert. They pay £l.35 each. How much ticket money is collected?* ➢ *Programmes cost 15p each. Selling programmes raises £12.30. How many programmes are sold?*

Reasoning opportunities and probing questions	Mathematical Language	Possible misconceptions
• Provide examples of multiplication of two-digit and three-digit numbers by a one-digit number using formal written layout with missing digits. Challenge pupils to find these digits and explain their reasoning. • A × B × C = 120. Show me possible values for A, B and C. And another triple. And another triple. • Always/Sometimes/Never: *'When you multiply two number together, the answer is greater than both of the two numbers'.* • NCETM: Multiplication and Division Reasoning	Mental arithmetic Place value Multiply, Multiplication, Times, Product Commutative Divide, Division Tenth, Hundredth Factor, Factor pairs Short multiplication Operation Estimate	• Some pupils may write statements such as 2 ÷ 8 = 4 • Some pupils may carry the wrong digit when using short multiplication; for example: 3 4 5 × 3 1 5

Fractions including decimals

Key concepts (National Curriculum statements)
Pupils should be taught to:

- recognise and show, using diagrams, families of common equivalent fractions
- count up and down in hundredths; recognise that hundredths arise when dividing an object by one hundred and dividing tenths by ten.
- solve problems involving increasingly harder fractions to calculate quantities, and fractions to divide quantities, including non-unit fractions where the answer is a whole number
- add and subtract fractions with the same denominator
- recognise and write decimal equivalents of any number of tenths or hundredths
- recognise and write decimal equivalents to 1/4, ½, ¾
- find the effect of dividing a one- or two-digit number by 10 and 100, identifying the value of the digits in the answer as ones, tenths and hundredths
- round decimals with one decimal place to the nearest whole number
- compare numbers with the same number of decimal places up to two decimal places
- solve simple measure and money problems involving fractions and decimals to two decimal places.

Notes and guidance (non-statutory)

- Pupils should connect hundredths to tenths and place value and decimal measure. They extend the use of the number line to connect fractions, numbers and measures.
- Pupils understand the relation between non-unit fractions and multiplication and division of quantities, with particular emphasis on tenths and hundredths.
- Pupils make connections between fractions of a length, of a shape and as a representation of one whole or set of quantities. Pupils use factors and multiples to recognise equivalent fractions and simplify where appropriate
- Pupils continue to practise adding and subtracting fractions with the same denominator, to become fluent through a variety of increasingly complex problems beyond one whole.
- Pupils are taught throughout that decimals and fractions are different ways of expressing numbers and proportions.
- Pupils' understanding of the number system and decimal place value is extended at this stage to tenths and then hundredths. This includes relating the decimal notation to division of whole number by 10 and later 100.
- They practise counting using simple fractions and decimals, both forwards and backwards.
- Pupils learn decimal notation and the language associated with it, including in the context of measurements. They make comparisons and order decimal amounts and quantities that are expressed to the same number of decimal places. They should be able to represent numbers with one or two decimal places in several ways, such as on number lines.

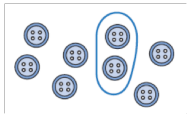
Possible Themes	Key learning points
• Develop knowledge of place value • Explore decimals • Investigate fractions and decimals • Calculate with fractions • Investigate equivalent fractions	• Recognise that hundredths arise from dividing a number or object into one hundred equal parts • Write hundredths as a fraction and as a decimal • Write decimal equivalents of any number of tenths and hundredths • Count up in hundredths • Count down in hundredths • Divide a one-digit number by 10 • Divide a one-digit number by 100 • Divide a two-digit number by 10 • Divide a two-digit number by 100 • Know and use the decimal equivalents to 1/4, 1/2, ¾ • Add fractions with the same denominator within and beyond one whole • Subtract fractions with the same denominator within and beyond one whole • Calculate a unit fraction of an amount when the answer is a whole number • Calculate a non-unit fraction of an amount when the answer is a whole number • Identify equivalent fractions from diagrams • Find families of equivalent fractions • Create diagrams to show families of equivalent fractions • Solve problems with increasingly harder fractions to calculate quantities
Prerequisite	NCETM – Ready to Progress

- ✓ Recognise and use tenths
- ✓ Divide one digit numbers by 10
- ✓ Add and subtract fractions with the same denominator within one whole
- ✓ Calculate fractions such as $^1/_2$ of 6 = 3
- ✓ Understand the concept of equivalent fractions
- ✓ Recognise equivalent fractions from diagrams
- ✓ Complete diagrams to show equivalent fractions

recognise and show, using diagrams, families of common equivalent fractions

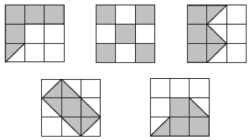
- Recognise that five tenths (5/10) or one half is shaded.

- Recognise that two eighths (2/8) or one quarter (¼) of the set of buttons is ringed



- Recognise that one whole is equivalent to two halves, three thirds, four quarters… For example, build a fraction 'wall' using a computer program and then estimate parts.
- Recognise patterns in equivalent patterns, such as:
- ½ = 2/4 = 3/6 = 4/8 = 5/10 = 6/12 = 7/14 And similar patterns for ⅓, ¼, ⅕, ⅙, 1/10.
- Here is a square. What fraction of the square is shaded?

- Here are five diagrams. Look at each one. Put a tick (✓) on the diagram is exactly ½ of it is shaded. Put a cross (X) if it is not.



count up and down in hundredths; recognise that hundredths arise when dividing an object by a hundred and dividing tenths by ten

Respond to questions such as:

- What does the digit 6 in 3.64 represent? The 4? What is the 4 worth in the number 7.45? The 5?
- Write the decimal fraction equivalent to:
- two tenths and five hundredths; twenty-nine hundredths; fifteen and nine hundredths.
- Continue the count 1.91, 1.92, 1.93, 1.94 ...
- Suggest a decimal fraction between 4.1 and 4.2
- Know how many 10 pence pieces equal a pound, how many 1 pence pieces equal a pound, how many centimetres make a metre.

recognise and write decimal equivalents of any number of tenths or hundredths

Recognise that, for example:

- 0.07 is equivalent to 7/100 6.35 is equivalent to 6 35/100
- Particularly in the contexts of money and measurement
- Respond to questions such as:
- Which of these decimals is equal to 19/100? 1.9 10.19 0.19 19.1 Write each of these as a decimal fraction: 27/100 3/100 2 33/100

recognise and write decimal equivalents to ¼; ½; ¾

Know that, for example

- 0.5 is equivalent to ½, 0.25 is equivalent to ¼, 0.75 is equivalent to ¾, 0.1 is equivalent to 1/10
- Particularly in the context of money and measurement.

find the effect of dividing a one- or two-digit number by 10 and 100, identifying the value of the digits in the answer as units, tenths and hundredths

Understand that:

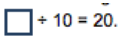
- When you divide a number by 1/100, the digits move one/two places to the right.
- Write a two-digit number on the board. Keep dividing by 10 and record the answer.

26
2.6
0.26
0.026

Describe the pattern.

- Respond to oral or written questions such as:
- How many times larger is 2600 than 26?
- How many £1 notes are in £120, £1200?
- Divide three hundred and ninety by ten.

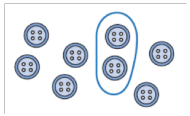
Write in the missing number



recognise and show, using diagrams, families of common equivalent fractions

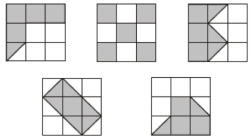
- Recognise that five tenths (5/10) or one half is shaded.

- Recognise that two eighths (2/8) or one quarter (¼) of the set of buttons is ringed



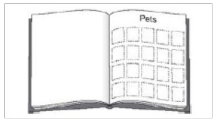
- Recognise that one whole is equivalent to two halves, three thirds, four quarters… For example, build a fraction ‘wall’ using a computer program and then estimate parts.
- Recognise patterns in equivalent patterns, such as:
- ½ = 2/4 = 3/6 = 4/8 = 5/10 = 6/12 = 7/14 And similar patterns for ⅓, ¼, ⅕, ⅙, 1/10.
- Here is a square. What fraction of the square is shaded?

- Here are five diagrams. Look at each one.

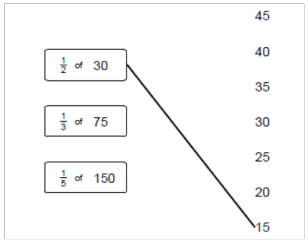


solve problems involving increasingly harder fractions to calculate quantities, and fractions to divide quantities, including non-unit fractions where the answer is a whole number

- What is one-fifth of twenty-five?
- Write the missing number to make this correct. $\frac{1}{4}$ of 24 = $\frac{1}{2}$ of $\square$
- Mary has 20 pet stickers to go on this page.
  ¼ of them are dog stickers. ½ of them are cat stickers. The rest are rabbit stickers. How many rabbit stickers does she have?



- Match each box to the correct number. One has been done for you.



Reasoning opportunities and probing questions	Mathematical Language	Possible misconceptions
• Jenny is counting in hundredths '…. 2.07, 2.08, 2.09, 2.010, 2.011 …'. Do you agree with Jenny? Explain your answer. • Convince me 0.17 = $\frac{17}{100}$ • Kenny thinks the decimal 0.8 can be written as $\frac{80}{100}$. Lenny thinks the decimal 0.8 can be written as $\frac{8}{10}$. Who do you agree with? Explain your thinking. • Always/Sometimes/Never: To divide a number by 10, remove the zero. • Convince me $\frac{1}{4}$ = 0.25 • NCETM: Fractions Reasoning • Show me a fraction equivalent to $\frac{3}{4}$ that no one else will think of. And another. And another • Show me two fractions that add together to make a whole. And another pair. And another pair. • Show me one improper and one proper fraction such that the difference is less than one whole. And another pair. And another pair. • Kenny thinks that ¾ of 24 is 2 because 24 ÷ 4 = 6 and 6 ÷ 3 = 2. Do you agree with Kenny? Explain your answer. • Jenny think that $\frac{16}{64}$ = $\frac{1}{4}$ because your can simply cancel the '6' in the numerator and denominator. Do you agree with Jenny's method for finding equivalent fractions? Explain your answer	Place value Tenth, hundredth Decimal Divide Fraction Numerator Denominator Tenth Hundredth Decimal Fraction Unit fraction, non-unit fraction Improper fraction Top-heavy fraction Numerator, denominator Add, subtract Equivalent (fraction) Family	• Some pupils may think that the first place value heading after the decimal point is 'one-ths' /'unit-ths' and the second place value heading is 'tenths'. • Some pupils may read 0.25 as 'nought point twenty five' • Some pupils may think that $^1/_4$ = 1.4 • Some pupils may think that you simply add/subtract the numerators and add/subtract the denominators when adding fractions. • Some pupils may think that you find the non-unit fraction of an amount by dividing by the denominator (as with unit fractions) and then dividing by the numerator. They do not make the connection that ¾ = 3 x ¼. • Some pupils may think that diagrams to show fractions must always be circular.

Measurement

Key concepts (National Curriculum statements)
Pupils should be taught to:

- Convert between different units of measure [for example, kilometre to metre; hour to minute]
- measure and calculate the perimeter of a rectilinear figure (including squares) in centimetres and metres
- find the area of rectilinear shapes by counting squares
- estimate, compare and calculate different measures, including money in pounds and pence

Notes and guidance (non-statutory)

- read, write and convert time between analogue and digital 12- and 24-hour clocks
- solve problems involving converting from hours to minutes; minutes to seconds; years to months; weeks to days.

Possible Themes	Key learning points
• Solve problems involving time • Solve problems involving money • Estimate measures • Convert between measures • Solve problems involving measurement • Solve problems involving money • Calculate perimeter • Investigate area	• Read digital 24-hour clocks • Write times using digital 24-hour clock • Write times using analogue 12-hour clock • Convert between 12-hour time and 24-hour notation • Solve problems involving converting from hours to minutes and minutes to seconds; • Solve problems involving converting from weeks to days • Solve problems involving converting from years to months • Solve problems involving decimal notation to record money • Convert between kilometres and metres • Convert between centimetres and millimetres • Convert between kilograms and grams • Convert between litres and millilitres • Solve measurement problems involving fractions • Solve money problems involving fractions • Solve measurement problems involving decimals to two decimal places • Solve money problems involving decimals to two decimal places • Measure and calculate the perimeter of 2D shapes when dimensions are unknown • Calculate the perimeter of rectangles (including squares) when dimensions are known • Calculate the perimeter of other rectilinear shapes when dimensions are known • Find the area of rectangles (including squares) by counting squares • Find the area of other rectilinear shapes by counting squares • Solve problems involving perimeter • Solve problems involving area
Prerequisite	NCETM – Ready to Progress

✓ Use analogue and digital 12-hour clocks ✓ Know the number of seconds in a minute, minutes in an hour, hours in a day, and the number of days in each week, month, year and leap year ✓ Know the value of all British coins and notes ✓ Know the number of pence in a pound ✓ Calculate the duration of time for a given event or task ✓ Use a ruler to measure lengths to the nearest millimetre ✓ Use digital and mechanical scales to measure mass ✓ Use measuring vessels to measure a volume of liquid ✓ Choose appropriate units to state the result of a measurement ✓ Compare the length (mass, volume, capacity) of two or more objects ✓ Solve measurement problems involving addition or subtraction ✓ Find the perimeter of a simple 2D shape by measuring	estimate, compare and calculate different measures, including money in pounds and pence ➢ Draw on their calculation strategies to solve one- and two-step word problems, including those involving money and measures. They use rounding to estimate the solution, choose an appropriate method of calculation (mental, mental with jottings, written method) and then check to see whether their answer seems sensible. They throw a beanbag three times and find the difference between their longest and shortest throws. After measuring their height, they work out how much taller they would have to grow to be the same height as their teacher. They solve problems such as: o Dad bought three tins of paint at £5.68 each. How much change does he get from £20? o A family sets off to drive 524 miles. After 267 miles, how much further do they still have to go? o Tins of dog food cost 42p. They are put into packs of 10. How much does one pack of dog food cost? 10 packs? o A can of soup holds 400 ml. How much do 5 cans hold? Each serving is 200 ml. How many cans would I need for servings for 15 people? o I spent £4.63, £3.72 and 86p. How much did I spend altogether? o A string is 6.5 metres long. I cut off 70 cm pieces to tie up some balloons. How many pieces can I cut from the string? o A jug holds 2 litres. A glass holds 250 ml. How many glasses will the jug fill? o Dean saves the same amount of money each month. He saves £149.40 in a year. How much money does he save each month? read, write and convert time between analogue and digital 12- and 24-hour clocks solve problems involving converting from hours to minutes; minutes to seconds; years to months; weeks to days. ➢ Solve problems involving units of time, explaining and recording how the problem was solved. For example: Raiza got into the pool at 2:26 pm. She swam until 3 o'clock. How long did she swim? They count on to find the difference between two given times, using a number line or time line where appropriate and use the 24-hour clock to measure time. Convert between different units of measure [for example, kilometre to metre; hour to minute] ➢ Learn the relationships between familiar units of measurement. They learn that kilo means one thousand to help them remember that there are 1000 grams in 1 kilogram and 1000 metres in 1 kilometre. They respond to questions such as: A bag of flour weighs 2 kg. How many grams is this? They suggest suitable units to measure length, weight and capacity; for example, they suggest a metric unit to measure the length of their book, the weight of a baby, the capacity of a mug. They suggest things that you would measure in kilometres, metres, litres, kilograms, etc. ➢ Record lengths using decimal notation, for example recording 5 m 62 cm as 5.62 m, or 1 m 60 cm as 1.6 m. They identify the whole-number, tenths and hundredths parts of numbers presented in decimal notation and relate the whole number, tenths and hundredths parts to metres and centimetres in length. estimate, compare and calculate different measures, including money in pounds and pence ➢ Draw on their calculation strategies to solve one- and two-step word problems, including those involving money and measures. They use rounding to estimate the solution, choose an appropriate method of calculation (mental, mental with jottings, written method) and then check to see whether their answer seems sensible. They throw a beanbag three times and find the difference between their longest and shortest throws. After measuring their height, they work out how much taller they would have to grow to be the same height as their teacher. They solve problems such as: o Dad bought three tins of paint at £5.68 each. How much change does he get from £20? o A family sets off to drive 524 miles. After 267 miles, how much further do they still have to go? o Tins of dog food cost 42p. They are put into packs of 10. How much does one pack of dog food cost? 10 packs? o A can of soup holds 400 ml. How much do 5 cans hold? Each serving is 200 ml. How many cans would I need for servings for 15 people? o I spent £4.63, £3.72 and 86p. How much did I spend altogether? o A string is 6.5 metres long. I cut off 70 cm pieces to tie up some balloons. How many pieces can I cut from the string? o A jug holds 2 litres. A glass holds 250 ml. How many glasses will the jug fill? Dean saves the same amount of money each month. He saves £149.40 in a year. How much money does he save each month? • Learn the relationships between familiar units of measurement. They learn that kilo means one thousand to help them remember that there are 1000 grams in 1 kilogram and 1000 metres in 1 kilometre. • They respond to questions such as: A bag of flour weighs 2 kg. How many grams is this? They suggest suitable units to measure length, weight and capacity; for example, they suggest a metric unit to measure the length of their book, the weight of a baby, the capacity of a mug. They suggest things that you would measure in kilometres, metres, litres, kilograms, etc.

- Record lengths using decimal notation, for example recording 5 m 62 cm as 5.62 m, or 1 m 60 cm as 1.6 m. They identify the whole-number, tenths and hundredths parts of numbers presented in decimal notation and relate the whole number, tenths and hundredths parts to metres and centimetres in length.
- Measure the edges of a rectangle and then combine these measurements. They realise that by doing this they are calculating its perimeter. Given the perimeter of a rectangle they investigate what the lengths of its sides could be. They work out the perimeter of irregular shapes drawn on a centimetre square grid, e.g. using the ITP 'Area'.

- For example, they draw irregular shapes on centimetre square grids, and compare their areas and perimeters.
- They use rounding to estimate the solution, choose an appropriate method of calculation (mental, mental with jottings, written method) and then check to see whether their answer seems sensible. They throw a beanbag three times and find the difference between their longest and shortest throws. After measuring their height, they work out how much taller they would have to grow to be the same height as their teacher.

They solve problems such as:

- A family sets off to drive 524 miles. After 267 miles, how much further do they still have to go?
- A can of soup holds 400 ml. How much do 5 cans hold? Each serving is 200 ml. How many cans would I need for servings for 15 people?
- A string is 6.5 metres long. I cut off 70 cm pieces to tie up some balloons. How many pieces can I cut from the string?

A jug holds 2 litres. A glass holds 250 ml. How many glasses will the jug fill?

Reasoning opportunities and probing questions

- Compare quantities by estimation
- Ask students to tell the time of using images of the Greenwich Observatory Clock or 24 Hour watches. How does a 24-hour watch need to operate differently to a 12-hour watch?
- Convince me that 18:40 is the same as 6:40 pm.
- Convince me that 'two pounds and five pence' is not written as '£2.5p'

- NCETM: Measurement Reasoning

- Show me a length in cm that is between 25mm and 0.5m. And another. And another.
- Which is greater, 4.5 litres or 460 ml? Explain your answer.
- Kenny thinks that 120 minutes is the same as 1 hour and 20 minutes. Do you agree with Kenny? Explain your answer.
- Convince me that 15:00 = 3 p.m.

- Always / Sometimes / Never: perimeter is given by multiplying the shortest side by 2, the longest side by 2, and then adding the results
- Jenny thinks that the perimeter of this rectangle is 9cm. Do you agree with Jenny?

- Create a shape with a perimeter greater then 30 m

Mathematical Language

Analogue
Digital
12-hour
24-hour
Second, Minute, Hour
Day, Week, Month, Year
Pound (£)
Pence (p)
Length
Mass
Volume
Length, distance
Mass
Volume
Capacity
Metre, centimetre, millimetre
Kilogram, gram
Litre, millilitre
Hour, minute, second
Decimal
Perimeter
Area
Dimensions
Square
Rectangle
Rectilinear
Polygon
Millimetre, Centimetre, Metre, Kilometre

Possible misconceptions

- Some pupils do not write 24-hour times as four digits, for example 1:50 rather than 01:50 for 1.50 a.m.
- Some pupils may write 'one pound and eight pence' as '£1.8'
- Some pupils may use both '£' and 'p' symbols, for example '£4.56p'
- Some pupils may write 'four pounds and fifty six pence' as '4.56p' or '£456'
- Some pupils may write amounts of money incorrectly; e.g. £3.5 for £3.50, especially if a calculator is used at any point
- Some pupils may apply an incorrect understanding that there are 100 minutes in a hour when solving problems
- Some pupils may struggle when converting between 12- and 24-hour clock notation; e.g. thinking that 15:00 is 5 o' clock
- Some pupils may find perimeters by counting the squares that are around the outside of the shape, rather than the associated lengths. This will result in an answer four less than the correct answer if working with rectangles.
- Some pupils may confuse the concepts of area and perimeter
- Some pupils may think that perimeter is always given by 2(a + b) or 2a + 2b, but this is only the case for certain 2D shapes
- Some pupils may think that you multiply the numbers to find the perimeter of a shape.
- Some pupils may think that you cannot find the perimeter of a shape unless all the dimensions are given.
- Some pupils may just add the given dimensions, rather than consider any unlabelled dimensions

Geometry

Key concepts (National Curriculum statements)
Pupils should be taught to:

- compare and classify geometric shapes, including quadrilaterals and triangles, based on their properties and sizes
- identify acute and obtuse angles and compare and order angles up to two right angles by size
- identify lines of symmetry in 2-D shapes presented in different orientations
- complete a simple symmetric figure with respect to a specific line of symmetry.
- describe positions on a 2-D grid as coordinates in the first quadrant
- describe movements between positions as translations of a given unit to the left/right and up/down
- plot specified points and draw sides to complete a given polygon.

Notes and guidance (non-statutory)

- Pupils continue to classify shapes using geometrical properties, extending to classifying different triangles (for example, isosceles, equilateral, scalene) and quadrilaterals (for example, parallelogram, rhombus, trapezium).
- Pupils compare and order angles in preparation for using a protractor and compare lengths and angles to decide if a polygon is regular or irregular.
- Pupils draw symmetric patterns using a variety of media to become familiar with different orientations of lines of symmetry; and recognise line symmetry in a variety of diagrams, including where the line of symmetry does not dissect the original shape.
- Pupils draw a pair of axes in one quadrant, with equal scales and integer labels. They read, write and use pairs of coordinates, for example (2, 5), including using coordinateplotting ICT tools.

Possible Themes	Key learning points
• Explore symmetry • Explore symmetrical patterns • Investigate 2D shapes • Develop knowledge of angles • Compare angles • Understand and use Cartesian coordinates • Use transformations to move shapes	• Identify and describe an equilateral triangle • Identify and describe an isosceles triangle • Identify and describe a scalene triangle • Identify and describe a parallelogram • Identify and describe a rhombus • Identify and describe a trapezium • Identify and describe a kite • Classify 2D shapes • Identify lines of symmetry of a 2D shape • Identify a line of symmetry of a pattern and for a diagram of a reflection • Use a line of symmetry to produce a symmetrical pattern • Use a line of symmetry to complete a symmetrical shape • Identify acute angles • Identify obtuse angles • Identify acute angles in shapes • Identify obtuse angles in shapes • Identify right angles in shapes • Compare angles up to two right angles in size • Order angles up to two right angles in size • Use coordinates to describe the position of a point in the first quadrant • Plot points in the first quadrant using co-ordinates • Use coordinates to plot a set of points to construct a polygon • Solve problems involving coordinates • Describe movements between positions as translations of a given unit to the left/right • Describe movements between positions as translations of a given unit to the up/down • Describe movements between positions as translations of a given unit to the left/right and up/down • Solve problems involving translation

Prerequisite	NCETM – Ready to Progress

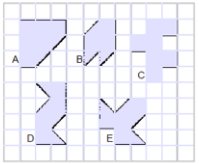
- ✓ Reflect a shape in a vertical line of symmetry
- ✓ Use a ruler to construct a straight line joining two points
- ✓ Know the names of special quadrilaterals
- ✓ Understand angles as a measure of turn
- ✓ Recognise angles in shapes
- ✓ Identify right angles as a quarter turn
- ✓ Know names and basic properties of polygons
- ✓ Know the language of movement; left, right, up and down

compare and classify geometric shapes, including quadrilaterals and triangles, based on their properties and sizes

- ➢ Pupils should be able to complete this sentence:
  All equilateral triangles have …

identify lines of symmetry in 2-D shapes presented in different orientations

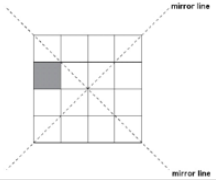
Here are five shapes on a square grid.



- ➢ Write the letters of the two shapes which have a line of symmetry.

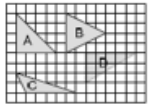
complete a simple symmetric figure with respect to a specific line of symmetry

Here is a shaded square on a grid. Shade in 3 more squares so that the design is symmetrical in both mirror lines.



identify acute and obtuse angles and compare and order angles up to two right angles by size

Here are four triangles drawn on a square grid.

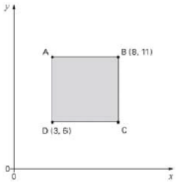


Write the letter for each triangle in the correct region of the sorting diagram. One has been done for you.

	has a right angle	has an obtuse angle	has an acute angle
is isosceles	A		
is not isosceles			

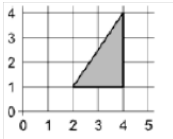
describe positions on a 2-D grid as coordinates in the first quadrant

Here is a shaded square.

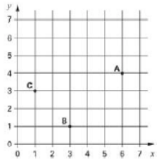


Write the coordinates for point A.

describe movements between positions as translations of a given unit to the left/right and up/down

I can describe where a shape will be after translation	This triangle is translated two squares to the left and one square down. Give the coordinates of its vertices in the new position.	

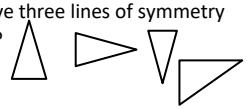
plot specified points and draw sides to complete a given polygon



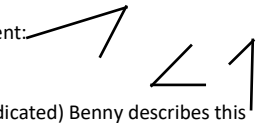
A, B and C are three corners of a rectangle. What are the coordinates of the fourth corner?

Reasoning opportunities and probing questions

- Show me a shape with one line of symmetry, two lines of symmetry. And Another …
- Always/Sometimes/Never: Triangles have three lines of symmetry
- What is the same and what is different ?



- Convince me that a rectangle does not have four lines of symmetry.
- Always/Sometimes/Never: Hexagons have six lines of symmetry
- NCETM: Geometry - Properties of Shapes Reasoning
- Show me an acute (obtuse) angle. And another. And another.
- (With your arms), show me an angle greater (less) than a quarter turn. And another. And another.
- Kenny thinks that a quarter turn is an acute angle. Jenny thinks that a quarter turn is an obtuse angle. Who is correct? Explain your answer.
- What is the same and what is different:



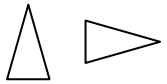
- (Given a grid with the point (2, 3) indicated) Benny describes this point as (2, 3). Jenny describes the point as (3, 2). Who do you agree with? Why?
- Convince me that the coordinates of the origin are (0, 0)
- Always / Sometimes / Never: A translation moves a shape further away from the origin.

Mathematical Language

Symmetry
Line of symmetry, Mirror line
Reflect, Reflection
Congruent
Perpendicular, Parallel
Vertex (Vertices)
Side, Edge
Quadrilateral
Square, Rectangle, Parallelogram, (Isosceles) Trapezium, Kite, Rhombus
Triangle
Scalene, Right-angled, Isosceles, Equilateral
Polygon, Hexagon, Pentagon, Octagon, Decagon
Circle
Turn
Angle
Right angle
Acute angle
Obtuse angle
Greater than, less than
2-D
Grid
Axis, axes, x-axis, y-axis
Origin
(First) quadrant
(Cartesian) coordinates
Point
Translation
Transformation
Left, right, up, down

Possible misconceptions

- Some pupils may think a rectangle has four lines of symmetry
- Some pupil may think that a triangle always has to have a horizontal base:



- Some pupils think that all hexagons, pentagons, octagons and decagons are regular.
- Some pupils think that a rhombus is a square ‘standing’ on one of its edges.
- Some pupils think that a quarter turn is an acute angle; some may think it’s an obtuse angle.
- Some pupils may think that one part of a drawn angle must be horizontal
- Some pupils may think that all turns have to be in a clockwise direction
- Some pupils may think that a drawn angle must use two lines of equal length
- When describing or carrying out a translation, some pupils may count the squares between the two shapes rather than the squares that describe the movement between the two shapes.
- Some pupils may muddle left and right
- Some pupils will confuse the order of x-coordinates and y-coordinates
- When constructing axes, some pupils may not realise the importance of equal divisions on the axes

Statistics

Key concepts (National Curriculum statements)
Pupils should be taught to:

- interpret and present discrete and continuous data using appropriate graphical methods, including bar charts and time graphs.
- solve comparison, sum and difference problems using information presented in bar charts, pictograms, tables and other graphs.

Notes and guidance (non-statutory)

- Pupils understand and use a greater range of scales in their representations.
- Pupils begin to relate the graphical representation of data to recording change over time.

Possible Themes	Key learning points
• Explore ways of presenting data • Solve problems involving charts and graphs	• Interpret a pictogram where the symbol represents multiple items • Interpret a bar chart • Interpret bar charts with different scales on the frequency axis • Create a bar chart with different scales on the frequency axis • Interpret a time graph • Create a time graph • Solve problems involving the data in charts and graphs • Solve problems involving the data in tables
Prerequisite	NCETM – Ready to Progress

✓ Interpret and construct a pictogram where the symbol represents multiple items ✓ Interpret and construct a simple bar chart where one centimetre represents 2, 5 or 10 items ✓ Interpret and construct tables of data	interpret and present discrete and continuous data using appropriate graphical methods, including bar charts and time graphs ➢ Collect data, measuring where necessary. They work with a range of data, such as shoe size and width of shoe across the widest part of the foot, the number of letters in children's names, the width of their hand spans, the distance around their neck and wrist, data from nutrition panels on cereal packets, and so on. ➢ They decide on a suitable question or hypothesis to explore for each data set they work on. For example, 'We think that…boys have larger shoes than girls', '…our neck measurements are twice as long as our wrist measurements', '…girls' names have more letters than boys' names' or '…children in our class would prefer to come to school by car but they usually have to walk'. ➢ Children consider what data to collect and how to collect it. They collect their data and organise it in a table. They choose a Venn or Carroll diagram, or a horizontal or vertical pictogram or bar chart to represent the data. Where appropriate, they use the support of an ICT package. They justify their choice within the group so that they can present it. ➢ They understand that they can join the tops of the bars on the bar-line chart to create a line graph because all the points along the line have meaning. solve comparison, sum and difference problems using information presented in bar charts, pictograms, tables and other graphs ➢ Undertake one or more of three enquiries: ○ What vehicles are very likely to pass the school gate between 10:00 am and 11:00 am? Why? What vehicles would definitely not pass by? Why not? What vehicles would be possible but not very likely? Why? What if it were a different time of day? What if the weather were different? ○ Does practice improve estimation skills? Children estimate the lengths of five given lines and record the estimate, measured length and difference. They repeat the activity with five more lines to see whether their estimation skills have improved after feedback. ○ What would children in our class most like to change in the school? Children carry out a survey after preliminary research to whittle down the number of options to a sensible number, e.g. no more than five. ➢ Children identify a hypothesis and decide what data to collect to investigate their hypothesis. They collect the data they need and decide on a suitable representation. In groups, they consider different possibilities for their representation and explain why they have made their choice. In the first enquiry, children use tallies and bar charts. In the second, they use tables and bar charts to compare the two sets of measurements. In the third, they use a range of tables and charts to show their results, including Venn and Carroll diagrams. They use ICT where appropriate.

Reasoning opportunities and probing questions	Mathematical Language	Possible misconceptions
• Show me a time graph of your day and tell me a story about it. And another. And another. • Penny draws a bar chart with horizontal bars. Benny says the bars must be vertical. Who is correct? Explain your answer. • Always/Sometimes/Never: One centimetre on the frequency axis of a bar chart represents one unit. • NCETM: Statistics Reasoning	Data Pictogram Symbol Key Tally Bar chart Time graph Scale Axis Graph Frequency	• Some pupils may interpret bar charts as one unit of frequency for each one square on the paper used. • Some pupils may not leave gaps between the bars in a bar chart • Some pupils may think that one centimetre on the frequency axis of a bar chart always represents one unit in a bar chart. • Some pupils may think that a symbol always represents one unit in a pictogram. • Some pupils may think that the bars of a bar chart must be vertical.

Year 5

Place Value

Key concepts (National Curriculum statements)

- identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers
- know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers
- establish whether a number up to 100 is prime and recall prime numbers up to 19
- recognise and use square numbers and cube numbers, and the notation for squared (^{2}) and cubed (^{3})
- solve problems involving multiplication and division including using their knowledge of factors and multiples, squares and cubes
- read Roman numerals to 1000 (M) and recognise years written in Roman numerals.

Notes and guidance (non-statutory)

- Pupils identify the place value in large whole numbers. They continue to use number in context, including measurement. Pupils extend and apply their understanding of the number system to the decimal numbers and fractions that they have met so far.
- They should recognise and describe linear number sequences, including those involving fractions and decimals, and find the term-to-term rule.
- They should recognise and describe linear number sequences

Possible Themes	Key learning points
• Identify multiples of numbers • Explore factors of numbers • Investigate prime numbers • Work with square and cube numbers • Work with numbers up to one million • Understand and use Roman numerals • Understand and use negative numbers	• Know and identify multiples of a given number • Know the identify factors of a given number • Find the 'common factor' of two numbers • Know the meaning of 'prime number' and recall the prime numbers less than 20 • Know the prime factors of a given number • Know how to test if a number up to 100 is prime • Know and identify square numbers • Know and identify cube numbers • Understand place value in numbers with up to seven digits • Order numbers up to and including those with seven digits • Write and read numbers up to and including those with seven digits • Read Roman numerals to 1000 (M) • Recognise years written in Roman numerals • Count forwards and backwards in whole number steps when negative numbers are included • Count forwards and backwards in whole number steps including through zero • Understand and use negative numbers in context, including temperatures below 0°C • Approximate any number by rounding to the nearest 10 000 or 100 000 • Approximate any number with two decimal place by rounding to the nearest whole number or rounding to one decimal place • Understand estimating as the process of finding a rough value of an answer or calculation • Estimate calculations with up to four digits
Prerequisite	NCETM – Ready to Progress

✓ Recall multiplication facts to 12 × 12 and associated division facts ✓ Recognise and use factor pairs and commutativity in mental calculations ✓ Understand and use place value in four-digit numbers ✓ Know Roman numerals from I to C ✓ Read numbers written in Roman numerals up to 100 ✓ Count forwards and backwards in whole number steps	Identify multiples and factors, including finding all factor pairs of a number, and common factors of 2 numbers know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers establish whether a number up to 100 is prime and recall prime numbers up to 19 ➢ Use the vocabulary factor, multiple and product. They identify all the factors of a given number; for example, the factors of 20 are 1, 2, 4, 5, 10 and 20. They answer questions such as: ➢ Find some numbers that have a factor of 4 and a factor of 5. What do you notice? ➢ My age is a multiple of 8. Next year my age will be a multiple of 7. How old am I? ➢ They recognise that numbers with only two factors are prime numbers and can apply their knowledge of multiples and tests of divisibility to identify the prime numbers less than 100. They explain that 73 children can only be organised as 1 group of 73 or 73 groups of 1, whereas 44 children could be organised as 1 group of 44, 2 groups of 22, 4 groups of 11, 11 groups of 4, 22 groups of 2 or 44 groups of 1. They explore the pattern of primes on a 100-square, explaining why there will never be a prime number in the tenth column and the fourth column. read, write, order and compare numbers to at least 1 000 000 and determine the value of each digit ➢ Explain what each digit represents in whole numbers and decimals with up to two places and partition, round and order these numbers. ➢ Answer problems such as o What is the value of the 7 in 3 274 105? o Write in figures forty thousand and twenty. o A number is partitioned like this: o 4 000 000 + 200 000 + 60 000 + 300 + 50 + 8 o Write the number. Now read it to me. o A car costs more than £8600 but less than £9100. Tick the prices that the car might cost. o £8569 □ £9090 □ £9130 □ £8999 □ ➢ count forwards or backwards in steps of powers of 10 for any given number up to ➢ 1 000 000 Count from any given number in powers of 10 and decimal steps extending beyond zero when counting backwards; relate the numbers to their position on a number line Answer problems such as: ➢ Write the next number in this counting sequence: 110 000, 120 000, 130 000 … ➢ Create a sequence that goes backwards and forwards in tens and includes the number 190. Describe your sequence. ➢ Here is part of a sequence: 30, 70, 110, □, 190, □. How can you find the missing numbers? interpret negative numbers in context, count forwards and backwards with positive and negative whole numbers, including through 0 ➢ Count from any given number in whole-number and decimal steps extending beyond zero when counting backwards; relate the numbers to their position on a number line. read Roman numerals to 1000 (M) and recognise years written in Roman numerals ➢ Recognise Roman numerals in their historical context Read and write Roman numerals to one thousand

Reasoning opportunities and probing questions	Mathematical Language	Possible misconceptions
• Show me a prime (square) number. And another. And another. • Kenny says '16 is a square number because $8^2 = 16$'. Explain why Kenny is wrong. • Convince me that 91 is not a prime number • Show me an example of a multiple of 4. And another. Now find a multiple of 4 that you think no one else in the room will choose. • Look at this number (1 029 628). Show me another number (with 4, 5, 6, 7 digits) that includes a 9 with the same value. And another. And another … • Jenny reads the number 1 029 008 as 'one million, twenty nine thousand and eight'. Kenny reads the same number as 'one million, two hundred and nine thousand and eight'. Who is correct? How do you know? • Convince me that 2014 is MMXIV in Roman numerals • Convince me that -17°C is colder than -14°C NCETM: Place Value Reasoning	Multiple (Common) factor Divisible Factor pairs Prime number, Composite number Square number, Cube number Power Place value Digit Roman numerals Negative number	• Many pupils believe that 1 is a prime number – a misconception which can arise if the definition is taken as 'a number which is divisible by itself and 1'. • Some pupils may think that 91 is a prime number as it follows a pattern 11, 31, 41, 61, 71, etc. • A common misconception is to believe that $6^2 = 6 \times 2 = 12$ • Some pupils think the fifth place value is 'millions' - eg 24 567 is two million, four thousand, five hundred and sixty seven. • Some pupils can confuse the language of large (and small) numbers since the prefix 'milli- means 'one thousandth' (meaning that there are 1000 millimetres in a metre for example) while one million is actually a thousand thousand. • The use of IIII on a clock face suggests that a Roman numeral can be repeated four times, but this is a special case. In general, three is the maximum number of repeats and the subtractive method should be used instead (i.e. IV)

Addition and Subtraction

Key concepts (National Curriculum statements)
Pupils should be taught to:

- add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction)
- add and subtract numbers mentally with increasingly large numbers
- use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy
- solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why.

Notes and guidance (non-statutory)

- Pupils practise using the formal written methods of columnar addition and subtraction with increasingly large numbers to aid fluency
- They practise mental calculations with increasingly large numbers to aid fluency (for example, 12 462 – 2300 = 10 162).

Possible Themes	Key learning points
• Develop mental addition and subtraction skills • Extend written methods of addition and subtraction • Solve problems involving addition and subtraction	• Add four-digit numbers and ones, tens and hundreds mentally • Add four-digit numbers and thousands mentally • Subtract four-digit numbers and ones, tens and hundreds mentally • Subtract four-digit numbers and thousands mentally • Use columnar addition for numbers with more than four digits with no carrying required • Use columnar addition for numbers with more than four digits with carrying required • Use columnar subtraction for numbers with more than four digits with no exchanging required • Use columnar subtraction for numbers with more than four digits with exchanging required
Prerequisite	NCETM – Ready to Progress

✓ Add and subtract numbers mentally, including a three-digit number and ones, tens or hundreds ✓ Use column addition and subtraction for numbers up to four digits ✓ Estimate the answer to a calculation	add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction) ➢ Children should be able to use standard written methods for addition and subtraction, e.g. calculate 14 136 + 3258 + 487 or 23 185 – 2078 ➢ Use written methods to find missing numbers in addition and subtraction calculations, e.g. 6432 + □ = 8025 ➢ Use written methods to add and subtract numbers with different numbers of digits, e.g. Find all the different totals that can be made using any three of these five numbers: 14 721, 76, 9534, 788, 6 add and subtract numbers mentally with increasingly large numbers ➢ Children should be able to respond rapidly to oral or written questions, explaining the strategy used, ➢ e.g. 750 take away 255, take 400 from 1360, 4500 minus 1050, subtract 3250 from 7600, 1800 less than 3300, 4000 less than 11 580 ➢ Derive quickly related facts, ➢ e.g. 80 + 50 = 130, 130 – 50 = 80, 800 + 500 = 1300, 1300 – 800 = 500 ➢ Derive quickly number pairs that total 100 or pairs of multiples of 50 that total 1000, ➢ e.g. 32 + 68 = 100 or 150 + 850 = 1000 ➢ Identify and use near doubles, ➢ e.g. work out 28 + 26 = 54 by doubling 30 and subtracting first 2, then 4, or by doubling 26 and adding 2 ➢ Add or subtract the nearest multiple of 10, 100 or 1000 and adjust, ➢ e.g. adding or subtracting 9, 19, 29 ... to/from any two-digit number ➢ Work out mentally by counting up from a smaller to a larger number e.g. 8000 – 2785 is 5 + 10 + 200 + 5000 = 5215 ➢ Understand and use language associated with addition and subtraction, e.g. difference, sum, total solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why ➢ Children should be able to choose the appropriate operations to solve multi-step problems, decide whether the calculations can be done mentally or using a written method and explain and record how the problem was solved using numbers, signs and symbols. ➢ e.g. 13 502 people were at the match last week and there are 2483 more this week, how many more people need to attend to bring the total to the club's target of 20 000 people? ➢ Identify and obtain the necessary information to solve the problem and determine if there is any important information missing, e.g. calculating total cost of a holiday for a family, given prices for adults and children and surcharges for particular resorts.

Reasoning opportunities and probing questions	Mathematical Language	Possible misconceptions
• Provide examples of column addition and subtraction with missing digits. Challenge pupils to find these digits and explain their reasoning. • Show me an example of a column addition (that includes carrying) with the answer 54192 • Convince me that 56095 – 23622 = 32473 • NCETM: Addition and Subtraction Reasoning	Addition Subtraction Sum, Total Difference, Minus, Less Column addition Column subtraction Exchange Operation Estimate	• When subtracting mentally some pupils may deal with columns separately and not combine correctly; e.g. 180 – 24: 180 – 20 = 160. Taking away 4 will leave 6. So the answer is 166. • Some pupils incorrectly assume and use commutativity within column subtraction; for example: 7 4 1 2 6 – 2 3 7 3 4 5 1 6 1 2 • Some pupils may not use place value settings correctly (especially when the numbers have a different number of digits)

Multiplication and Division

Key concepts (National Curriculum statements)
Pupils should be taught to:

- identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers
- know and use the vocabulary of prime numbers, prime factors and composite (nonprime) numbers
- establish whether a number up to 100 is prime and recall prime numbers up to 19
- multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers
- multiply and divide numbers mentally drawing upon known facts
- divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context
- multiply and divide whole numbers and those involving decimals by 10, 100 and 1000

Notes and guidance (non-statutory)

- recognise and use square numbers and cube numbers, and the notations
- Solve problems involving multiplication and division including using their knowledge of factors and multiples, squares and cubes
- solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign
- solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates.

Possible Themes	Key learning points
• Develop mental arithmetic skills • Explore multiplication and division of decimals • Develop written methods of multiplication • Develop written methods of division • Solve problems involving multiplication and division	• Multiply a whole number by 10 • Multiply a whole number by 100 • Multiply a whole number by 1000 • Multiply a decimal by 10 • Multiply a decimal by 100 • Multiply a decimal by 1000 • Divide a whole number by 10 • Divide a whole number by 100 • Divide a whole number by 1000 • Divide a decimal by 10 • Divide a decimal by 100 • Divide a decimal by 1000 • Multiply numbers up to 4-digits by a one-digit number using short multiplication • Multiply three-digit numbers by a two-digit number using long multiplication • Multiply four-digit numbers by a two-digit number using long multiplication • Divide a three-digit number by a one-digit number using short division with no remainder • Divide a three-digit number by a one-digit number using short division with a remainder • Divide a four-digit number by a one-digit number using short division with no remainder • Divide a four-digit number by a one-digit number using short division with a remainder • Interpret a remainder appropriately for the context when carrying out division
Prerequisite	NCETM – Ready to Progress

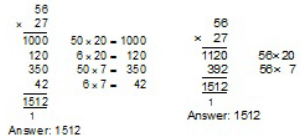
- ✓ Recall multiplication facts for multiplication tables up to 12 × 12
- ✓ Recall division facts for multiplication tables up to 12 × 12
- ✓ Find factor pairs of a given number
- ✓ Understand the commutativity of multiplication
- ✓ Multiply and divide a two-digit number by 10, 100
- ✓ Multiply a three-digit number by a one-digit number using short multiplication

multiply and divide numbers mentally, drawing upon known facts

- Rehearse multiplication facts and use these to derive division facts, to find factors of two-digit numbers and to multiply multiples of 10 and 100, e.g. 40 × 50. They use and discuss mental strategies for special cases of harder types of calculations, for example to work out 274 + 96,< 8006 – 2993, 35 × 11, 72 ÷ 3, 50 × 900. They use factors to work out a calculation such as 16 × 6 by thinking of it as 16 × 2 × 3. They record their methods using diagrams (such as number lines) or jottings and explain their methods to each other. They compare alternative methods for the same calculation and discuss any merits and disadvantages.

multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers

- Develop and refine written methods for multiplication. They move from expanded layouts (such as the grid method) towards a compact layout for HTU × U and TU × TU calculations. They suggest what they expect the approximate answer to be before starting a calculation and use this to check that their answer sounds sensible. For example, 56 × 27 is approximately 60 × 30 = 1800.



multiply and divide whole numbers and those involving decimals by 10, 100 and 1,000

- Recall quickly multiplication facts up to 10 × 10 and use them to multiply pairs of multiples of 10 and 100. They should be able to answer problems such as:
- the product is 400. At least one of the numbers is a multiple of 10. What two numbers could have been multiplied together? Are there any other possibilities?

divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context

- Extend written methods for division to include HTU ÷ U, including calculations with remainders. They suggest what they expect the approximate answer to be before starting a calculation and use this to check that their answer sounds sensible. They increase the efficiency of the methods that they are using. For example:
- 196 ÷ 6 is approximately 200 ÷ 5 = 40
- 3 2 r4 or 4/6 or 2/3
- Children know that, depending on the context, answers to division questions may need to be rounded up or rounded down. They explain how they decided whether to round up or down to answer problems such as:
- Egg boxes hold 6 eggs. A farmer collects 439 eggs. How many boxes can he fill
- Egg boxes hold 6 eggs. How many boxes must a restaurant buy to have 200 eggs?

solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign

solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates

- Use written methods to solve problems and puzzles such as:

Choose any four numbers from the grid and add them. Find as many ways as possible of making 1000.

275	382	81	174
206	117	414	262
483	173	239	138
331	230	325	170

- Place the digits 0 to 9 to make this calculation correct: □□□□ – □□□ = □□□.

Two numbers have a total of 1000 and a difference of 246. What are the two numbers?

Reasoning opportunities and probing questions	Mathematical Language	Possible misconceptions
• Find missing digits in otherwise completed long multiplication / short division calculations • Convince me that 247 × 12 = 2964 • What is the same and what is different: 1344 × 6 and 504 × 16? • What is wrong with this short division? How can you correct it? 0 10 7 r 5 8) 3 ^{8}6 ^{6}1 • NCETM: Multiplication and Division Reasoning	Multiply, Multiplication, Times, Product Commutative Divide, Division, Divisible Divisor, Dividend, Quotient, Remainder Factor Short multiplication, Long multiplication Short division Operation Estimate	• Some pupils may write statements such as 2 ÷ 8 = 4 • Some pupils may forget to '*put the zero down*' when multiplying the tens digit using long multiplication. • When using short division many pupils will at first struggle to deal correctly with any division where the divisor is greater than the first digit of the dividend; for example: 0 10 7 r 5 8) 3 ^{8}6 ^{6}1 • 3 ÷ 8 = 0 remainder 3, and so the 3 should be moved across. Instead, the 8 has been 'moved across' and therefore everything that follows has been correctly carried out based on an early misunderstanding.

Fractions, Decimals and Percentages

Key concepts (National Curriculum statements)
Pupils should be taught to:

- compare and order fractions whose denominators are all multiples of the same number
- identify, name and write equivalent fractions of a given fraction, represented visually, including tenths and hundredths
- recognise mixed numbers and improper fractions and convert from one form to the other and write mathematical statements > 1 as a mixed number
- add and subtract fractions with the same denominator and denominators that are multiples of the same number
- multiply proper fractions and mixed numbers by whole numbers, supported by materials and diagrams
- read and write decimal numbers as fractions
- recognise and use thousandths and relate them to tenths, hundredths and decimal equivalents
- round decimals with two decimal places to the nearest whole number and to one decimal place
- read, write, order and compare numbers with up to three decimal places
- solve problems involving number up to three decimal places
- recognise the per cent symbol (%) and understand that per cent relates to 'number of parts per hundred', and write percentages as a fraction with denominator 100, and as a decimal
- solve problems which require knowing percentage and decimal equivalents

Notes and guidance (non-statutory)

- Pupils should be taught throughout that percentages, decimals and fractions are different ways of expressing proportions.
- They extend their knowledge of fractions to thousandths and connect to decimals and measures.
- Pupils connect equivalent fractions > 1 that simplify to integers with division and other fractions > 1 to division with remainders, using the number line and other models, and hence move from these to improper and mixed fractions.
- Pupils connect multiplication by a fraction to using fractions as operators (fractions of), and to division, building on work from previous years. This relates to scaling by simple fractions, including fractions > 1.
- Pupils practise adding and subtracting fractions to become fluent through a variety of increasingly complex problems. They extend their understanding of adding and subtracting fractions to calculations that exceed 1 as a mixed number.
- Pupils continue to practise counting forwards and backwards in simple fractions.
- Pupils continue to develop their understanding of fractions as numbers, measures and operators by finding fractions of numbers and quantities.
- Pupils extend counting from year 4, using decimals and fractions including bridging zero, for example on a number line.
- Pupils say, read and write decimal fractions and related tenths, hundredths and thousandths accurately and are confident in checking the reasonableness of their answers to problems.
- They mentally add and subtract tenths, and one-digit whole numbers and tenths.
- They practise adding and subtracting decimals, including a mix of whole numbers and decimals, decimals with different numbers of decimal places, and complements of 1 (for example, 0.83 + 0.17 = 1).
- Pupils should go beyond the measurement and money models of decimals, for example, by solving puzzles involving decimals.
- Pupils should make connections between percentages, fractions and decimals

Possible Themes	Key learning points
• Explore the equivalence between fractions • Explore the equivalence between fractions and decimals • Understand the meaning of percentages • Explore mixed numbers • Calculate with fractions • Explore fractions, decimals and percentages	• Compare fractions whose denominators are multiples of the same number • Order fractions whose denominators are multiples of the same number • Identify equivalent fractions represented using tenths and hundredths • Understand and use thousandths • Write a number (less than1) with one decimal place as a fraction • Write a number (less than 1) with two decimal places as a fraction • Recognise that thousandths arise from dividing a number (or object) into one thousand equal parts and dividing hundredths by ten • Solve problems involving number up to three decimal places • Read a number with three decimal places • Compare and order a set of numbers written to three decimal places • Compare and order a set of numbers with a mixed number of decimal places • Understand that per cent relates to number of parts per hundred • Write any percentage as a fraction with a denominator of 100 • Write any percentage as a decimal • Convert a mixed number into an improper fraction (and vice versa) • Add fractions when one denominator is a multiple of the other including mixed numbers as part of the question and/or answer. • Subtract fractions when one denominator is a multiple of the other including mixed numbers as part of the question and/or answer • Multiply a proper fraction by a whole number • Multiply a mixed number by a whole number • Know percentage equivalents of $^1/_2$, $^1/_4$, $^1/_5$, $^2/_5$, $^4/_5$ and fractions with a denominator of 10 and 100 • Establish percentage equivalents of fractions with a denominator of 20, 25, 40 and 50 • Know decimal equivalents of $^1/_2$, $^1/_4$, $^1/_5$, $^2/_5$, $^4/_5$ and fractions with a denominator of 10 and 100 • Establish decimal equivalents of fractions with a denominator of 20, 25, 40 and 50
Prerequisite	NCETM – Ready to Progress

- ✓ Understand the concept of equivalent fractions
- ✓ Understand that tenths and hundredths can be written as fractions or as decimals
- ✓ Know that $^1/_4$ = 0.25, $^1/_2$ = 0.5 and $^3/_4$ = 0.75
- ✓ Understand the concept of an improper fraction
- ✓ Add and subtract fractions with the same denominator within and beyond one whole
- ✓ Recognise and use tenths and hundredths
- ✓ Understand that per cent relates to number of parts per hundred
- ✓ Understand that a percentage can be written as a fraction with a denominator of 100
- ✓ Write any percentage as a decimal

compare and order fractions whose denominators are all multiples of the same number

- Children should be able to circle the two fractions that have the same value, or choose which one is the odd one out and justify their decision.
  $^6/_{10}$, $^3/_5$, $^{18}/_{20}$, $^9/_{15}$

recognise mixed numbers and improper fractions and convert from one form to the other. Write mathematical statements >1 as a mixed number

- Put the correct symbol, < or >, in each box.
  3.03 ☐ 3.3
  0.37 ☐ 0.327
  Order these numbers: 0.27 0.207 0.027 2.07 2.7
- (e.g. ⅖ + ⅘ = $^6/_5$ = 1⅕)
- How many halves in: 1 ½ 3 ½ 9 ½ …?
- How many quarters in 1 ¼ 2 ¼ 5 ¼ ….?

multiply proper fractions and mixed numbers by whole numbers

- What is $^3/_{10}$ of: 50, 20, 100…?
- What is ⅘ of 50, 35, 100….?

read and write decimal numbers as fractions (e.g. 0.71 = $^{71}/_{100}$)

- What decimal is equal to 25 hundredths?
- Write the total as a decimal:
- 4 + $^6/_{10}$ + $^2/_{100}$ =
- Children partition decimals using both decimal and fraction notation, for example, recording 6.38 as 6 + $^3/_{10}$ + $^8/_{100}$ and as 6 + 0.3 + 0.08.

recognise and use thousandths and relate them to tenths, hundredths and decimal equivalents

- Recognise that
  0.007 is equivalent to $^7/_{1000}$
  6.305 is equivalent to $^{6305}/_{1000}$

read, write, order and compare numbers with up to three decimal places

- Write these numbers in order of size, starting with the smallest. 1.01, 1.001, 1.101, 0.11

solve problems involving numbers with up to three decimal places

$$\begin{array}{r} 72.8 \\ +\ \underline{54.6} \\ 127.4 \\ \underline{1} \end{array}$$

- 8 tenths add 6 tenths makes 14 tenths, or 1 whole and 4 tenths. The 1 whole is 'carried' into the units column and the 4 tenths is written in the tenths column

recognise the per cent symbol (%) and understand that per cent relates to 'number of parts per hundred'

- Write in the missing numbers. 30% of 60 is ☐
  30% of ☐ is 60

write percentages as a fraction with denominator 100, and as a decimal

- Shade 10% of this grid.

- Which is bigger: 65% or ¾? How do you know?
- What percentage is the same as $^7/_{10}$? Explain how you know?
- What is $^{31}/_{100}$ as a percentage?

Which is a better mark in a test: 61% , or 30 out of 50? How do you know?

recognise mixed numbers and improper fractions and convert from one form to the other. Write mathematical statements >1 as a mixed number

- Put the correct symbol, < or >, in each box.
- 3.03 ☐ 3.3
  0.37 ☐ 0.327
  Order these numbers: 0.27 0.207 0.027 2.07 2.7

	• (e.g. ⅖ + ⅘ = $^{6}/_{5}$ = 1⅕) • How many halves in: 1 ½ 3 ½ 9 ½ …? • How many quarters in 1 ¼ 2 ¼ 5 ¼ ….? multiply proper fractions and mixed numbers by whole numbers • What is $^{3}/_{10}$ of: 50, 20, 100…? • What is ⅘ of 50, 35, 100….? read and write decimal numbers as fractions (e.g. 0.71 = $^{71}/_{100}$) • What decimal is equal to 25 hundredths? • Write the total as a decimal: • 4 + $^{6}/_{10}$ + $^{2}/_{100}$ = • Children partition decimals using both decimal and fraction notation, for example, recording 6.38 as 6 + $^{3}/_{10}$ + $^{8}/_{100}$ and as 6 + 0.3 + 0.08. recognise and use thousandths and relate them to tenths, hundredths and decimal equivalents Recognise that 0.007 is equivalent to $^{7}/_{1000}$ 6.305 is equivalent to $^{6305}/_{1000}$

Reasoning opportunities and probing questions	Mathematical Language	Possible misconceptions
• Show me a fraction that is equivalent to 7/10. And another … • Convince me that 6/8 is greater than 7/16 • Jenny says that 0.127 is 'one hundred and twenty seven thousandths'. Kenny says that 0.127 is 'one tenth, two hundredths and seven thousandths'. Who do you agree with? Explain your reasoning. • NCETM: Fractions Reasoning • Show me an improper fraction (mixed number). And another. • Kenny thinks that $\frac{1}{4} + \frac{2}{8} = \frac{3}{12}$. Explain why Kenny is incorrect. • Jenny thinks that you can only add or subtract fractions if they have the same common denominator. Do you agree with Jenny? Explain. • Show me a fraction, decimal and percentage 'equivalent family' (e.g. $\frac{1}{2}$ = 50% = 0.5). And another. And another … • Kenny thinks that $\frac{1}{3} \times 5 = \frac{5}{15}$. Do you agree with Kenny? Explain. • Convince me that $2\frac{2}{3} \times 3 = 8$ in at least 2 different ways.	Fraction Numerator Denominator Improper fraction, Proper fraction, Top-heavy fraction Tenth, hundredth, thousandth Per cent, Percentage Decimal Equivalent	• Some pupils may read 0.234 as 'nought point two hundred and thirty four'. This leads to the common misconception that, for example, 0.400 is a number larger than 0.76 • Pupils may not make the connection that a percentage is a different way of describing a proportion • Some pupils may think that equivalent fractions are found using an additive relationship rather than a multiplicative one: for example, that the fraction 4/5 is equivalent to 6/8 • Some pupils may think that you simply add the numerators and add the denominators when adding fractions. • Some pupils may think that you simply subtract the numerators and subtract the denominators when subtracting fractions. • Some pupils may think that you simply multiply both the numerator denominator when multiplying a fraction by a whole number. • Some pupils may think that you simply multiply the whole number and then the fraction when multiplying a mixed number by a whole number, e.g. $3\frac{3}{4} \times 2 = 6\frac{6}{4}$

Measurement

Key concepts (National Curriculum statements)
Pupils should be taught to:

- convert between different units of metric measure (for example, kilometre and metre; centimetre and metre; centimetre and millimetre; gram and kilogram; litre and millilitre)
- understand and use approximate equivalences between metric units and common imperial units such as inches, pounds and pints
- measure and calculate the perimeter of composite rectilinear shapes in centimetres and metres
- calculate and compare the area of rectangles (including squares), and including using standard units, square centimetres (cm2) and square metres (m2) and estimate the area of irregular shapes
- estimate volume [for example, using 1 cm3 blocks to build cuboids (including cubes)] and capacity [for example, using water]
- solve problems involving converting between units of time
- use all four operations to solve problems involving measure [for example, length, mass, volume, money] using decimal notation, including scaling

Notes and guidance (non-statutory)

- Pupils use their knowledge of place value and multiplication and division to convert between standard units.
- Pupils calculate the perimeter of rectangles and related composite shapes, including using the relations of perimeter or area to find unknown lengths. Missing measures questions such as these can be expressed algebraically, for example 4 + 2b = 20 for a rectangle of sides 2 cm and b cm and perimeter of 20cm.
- Pupils calculate the area from scale drawings using given measurements.
- Pupils use all four operations in problems involving time and money, including conversions (for example, days to weeks, expressing the answer as weeks and days).

Possible Themes	Key learning points
• Convert between measures • Know and work with common Imperial units • Solve problems involving measurement • Solve problems involving money • Exploring the perimeter of composite shapes • Calculate areas of rectangles • Investigate volume and capacity • Solve problems involving time • Interpret information in tables • Interpret information in timetables	• Convert between kilometres and metres • Convert between centimetres and metres • Convert between centimetres and millimetres • Convert between kilograms and grams • Convert between litres and millilitres • Use decimal notation when converting between metric units of length, mass and volume / capacity • Know approximate equivalencies between metric and imperial units • Solving problems involving measures, including money • Calculate the perimeter of composite rectilinear shapes • Calculate the area of a rectangles, including squares • Convert between square centimetres (cm^2) and square metres (m^2) • Estimate the area of irregular shapes bounded by straight lines • Estimate the area of irregular shapes that include curved lines • Estimate volume by using 1 cm^3 blocks to build cuboids, including cubes • Estimate capacity • Solve problems involvjng area and perimeter • Solve a problem involving converting between different units of time • Read and interpret information given in a table • Read and interpret information given in a timetable • Solve problems that involve interpreting timetables
Prerequisite	NCETM – Ready to Progress

- ✓ Convert between kilometres and metres, centimetres and millimetres
- ✓ Convert between litres and millilitres
- ✓ Convert between hours and minutes, minutes and seconds
- ✓ Use decimal notation to two decimal places when converting between measures
- ✓ Understand the concept of area
- ✓ Understand the concept of perimeter
- ✓ Calculate the perimeter of 2D shapes when dimensions are known
- ✓ Find the area of rectilinear shapes by counting squares
- ✓ Read, write and convert time between analogue and digital 12- and 24-hour clocks
- ✓ Know how to convert from hours to minutes; minutes to seconds; years to months; weeks to days

convert between different units of metric measure (for example, kilometre and metre; centimetre and metre; centimetre and millimetre; gram and kilogram; litre and millilitre

- ➢ What is two hundred and seventy six centimetres to the nearest metre?
- ➢ How many millimetres are in 3 centimetres?

understand and use approximate equivalences between metric units and common imperial units such as inches, pounds and pints

- ➢ This bag of sugar weighs 1kg. Approximately how many pounds (lb) of sugar would fit into another empty bag of the same size as this one? Tick the correct answer.

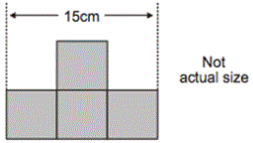
  20lb
  14lb
  2lb
  4lb

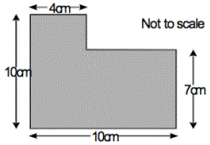
use all four operations to solve problems involving measure [for example, length, mass, volume, money] using decimal notation,

A day with Grandpa. Is an engaging problem using imperial units that challenges children's understanding of the concept of area rather than simply requiring them to follow a rule for finding areas of rectangles. These calculations should also help learners to see the advantages of the metric system as well as understand it more fully!

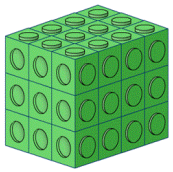
- What is two hundred and seventy six centimetres to the nearest metre?
- How many millimetres are in 3 centimetres?
- This bag of sugar weighs 1kg. Approximately how many pounds (lb) of sugar would fit into another empty bag of the same size as this one? Tick the correct answer.
  - 20lb
  - 14lb
  - 2lb
  - 4lb

- This shape is made from 4 shaded squares
-
- Calculate the perimeter of the shape





- Calculate the area of a rectangle which is eleven metres long by 5 metres wide.
- Which has the greatest area – a square with sides 6 cm long or a rectangle which is 7 cm long by 5 cm? How much greater is the area?
- estimate volume [for example, using 1 cm3 blocks to build cuboids (including cubes)] and capacity [for example, using water]



Fitting it in is an activity to fill cuboid shapes with multilink cubes. It ends with a 'create' challenge that will test children's knowledge in this area

solve problems involving converting between units of time

- ➢ 5 on the clock is a problem that requires children to be able to convert between 12 and 24 hour clocks confidently.

complete, read and interpret information in tables, including timetables

- I can find the information in a table or graph to answer a question
- The table shows the cost of coach tickets to different cities.

What is the total cost for a return journey to York for one

		Hull	York	Leeds
Adult	single	£12.50	£15.60	£10.25
	return	£23.75	£28.50	£19.30
Child	single	£8.50	£10.80	£8.25
	return	£14.90	£17.90	£14.75

Reasoning opportunities and probing questions

- Kenny thinks 1.5m = 105cm. Do you agree with Kenny? Explain your answer
- Show me an imperial (metric) unit of measure. And another. And another.
- Convince me that 3.07kg = 3070g.
- Which of the following is the best value for money?
  1 litre for £2 or 2 pints for £2
  5kg for 40p or 4lbs for 40p
  10cm for £2 or 5 inches for £2

- NCETM: Measurement Reasoning
- Jenny estimates the area of an irregular shape by counting all whole squares, and then matching up part squares to make whole squares. Benny estimates the area of the same shape by counting all whole squares and all squares that are mostly within the shape. He ignores squares mostly outside the shape. Whose method is best? Explain.
- Convince me that area of a rectangle = length × width
- Show me a shape with an area of 23 cm^2. And another, and another …
- (Using a timetable) I want to arrive in Chichester by 10:15. Show me a train that I could catch from Portsmouth. And another. What is the latest train I could catch? What time does this train leave Portsmouth?
- Convince me that that are 135 minutes between 1115 and 1:30 p.m.
- Jenny and Kenny are solving a problem that involves planning a journey. They are leaving Chester at 08:12. The journey takes 1 hour and 50 minutes. Jenny thinks that they will arrive at 09:62. Kenny thinks that they will arrive at 10:02. Who do you agree with? Explain your answer.

Mathematical Language

Length, distance
Mass, weight
Volume
Capacity
Metre, centimetre, millimetre
Kilogram, gram
Litre, millilitre
Hour, minute, second
Inch, foot, yard
Pound, ounce
Pint, gallon
Millennium
Century
Decade
Year
Month
Week
Day
Hour
Minute
Second
Timetable

Possible misconceptions

- Some pupils may apply incorrect beliefs about place value, such as 2.3 × 10 = 2.30.
- Many conversions within the metric system rely on multiplying and dividing by 1000. The use of centimetres as an 'extra unit' within the system breaks this pattern. Consequently there is a frequent need to multiply and divide by 10 or 100, and this can cause confusion about the connections that need to be applied.
- Some pupils may write amounts of money incorrectly; e.g. £3.5 for £3.50, especially if a calculator is used at any point
- Some pupils may confuse the concepts of area and perimeter
- Some pupils may think that you multiply the numbers to find the perimeter of a shape.
- Some pupils may think that you cannot find the perimeter of a shape unless all the dimensions are given.
- Some pupils may just add the given dimensions, rather than consider any unlabelled dimensions
  Some pupils may think that you multiply all the numbers to find the area of a rectangle
- Some pupils may write amounts of money incorrectly; e.g. £3.5 for £3.50, especially if a calculator is used at any point
- Some pupils may apply an incorrect understanding that there are 100 minutes in a hour when solving problems
- Some pupils may struggle when converting between 12- and 24-hour clock notation; e.g. thinking that 15:00 is 5 o' clock

Geometry

Key concepts (National Curriculum statements)
Pupils should be taught to:

- identify 3-D shapes, including cubes and other cuboids, from 2-D representations
- know angles are measured in degrees: estimate and compare acute, obtuse and reflex angles
- draw given angles, and measure them in degrees
- identify: angles at a point and one whole turn (total 360o)
- angles at a point on a straight line and ½ a turn (total 180o)
- other multiples of 90o
- use the properties of rectangles to deduce related facts and find missing lengths and angles
- distinguish between regular and irregular polygons based on reasoning about equal sides and angles.
- identify, describe and represent the position of a shape following a reflection or translation, using the appropriate language, and know that the shape has not changed.

Notes and guidance (non-statutory)

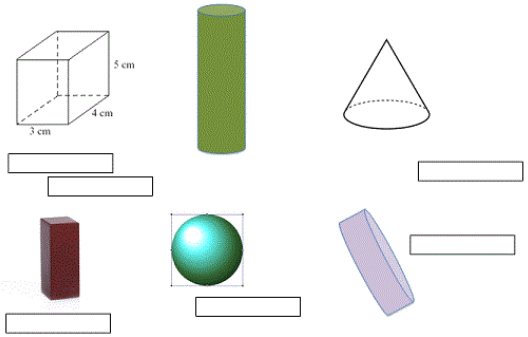
- Pupils become accurate in drawing lines with a ruler to the nearest millimetre, and measuring with a protractor. They use conventional markings for parallel lines and right angles.
- Pupils use the term diagonal and make conjectures about the angles formed between sides, and between diagonals and parallel sides, and other properties of quadrilaterals, for example using dynamic geometry ICT tools.
- Pupils use angle sum facts and other properties to make deductions about missing angles and relate these to missing number problems.

- Pupils recognise and use reflection and translation in a variety of diagrams, including continuing to use a 2-D grid and coordinates in the first quadrant. Reflection should be in lines that are parallel to the axes.

Possible Themes	Key learning points
• Investigate 3D shapes • Explore the properties of rectangles • Investigate polygons • Develop knowledge of angles • Measure angles • Draw angles	• Identify 3D-shapes from photographs and sketches • Identify 3D-shapes from nets • Identify 3D-shapes from diagrams on isometric paper • Construct diagrams of 3D-shapes on isometric paper • Use the properties of rectangles to find missing lengths and angles • Use the properties of rectangles to find points on a coordinate grid • Know the difference between a regular and an irregular polygon • Use the properties of regular polygons to find points on a coordinate grid • Know that angles are measured in degrees and estimate acute, obtuse and reflex angles • Know that a reflex angle is greater than 180° and estimate reflex angles • Identify and find angles at a point • Identify and find angles at a point on a straight line • Use a protractor to measure angles less than 180° • Use a protractor to measure angles greater than 180° • Use a protractor to draw angles less than 180° • Use a protractor to draw angles greater than 180° • Carry out a translation described using mathematical language • Describe a translation using mirror lines parallel to the axes • Carry out a reflection using a mirror line parallel to the axes • Carry out a reflection using a mirror line parallel to the axes and touching the object • Carry out a reflection using a mirror line parallel to the axes and crossing the object • Describe a reflection using mirror lines parallel to the axes • Understand that a translations and reflections produce a congruent image • Solve problems involving transformations
Prerequisite	NCETM – Ready to Progress

- ✓ Know the names of common 3D shapes
- ✓ Identify right angles
- ✓ Use coordinates in the first quadrant
- ✓ Understand that an acute angle is less than a right angle
- ✓ Understand that an obtuse angle is greater than a right angle and less than two right angles
- ✓ Identify acute angles
- ✓ Identify obtuse angles
- ✓ Identify acute, obtuse and right angles in shapes
- ✓ Compare angles up to two right angles in size
- ✓ Order angles up to two right angles in size
- ✓ Use coordinates in the first quadrant
- ✓ Describe a translation using mathematical language

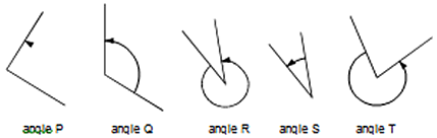
Identify 3-D shapes, including cubes and other cuboids, from 2D representations



These are pictures of 3D shapes. Which 3D shapes are pictured here? Put the names in the boxes.

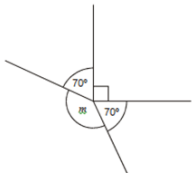
Know angles are measured in degrees: estimate and compare acute, obtuse and reflex angles

- ➢ Look at these angles. Label each angle acute, obtuse or reflex. List the 5 angles in order from smallest to largest.



- ➢ Draw given angles, and measure them in degrees (°)
- ➢ Measure A accurately. Use a protractor (angle measurer).

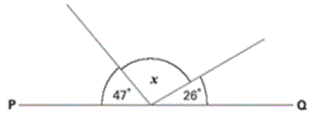
- ➢ Measure accurately the smallest angle in the above shape. Use a protractor (angle measurer).
- ➢ This diagram is not drawn accurately. Calculate the size of angle m



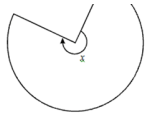
Identify:

- Angles at a point and one whole turn (total 360°)
- Angles at a point on a straight line and ½ a turn (total 180°)
- Other multiples of 90°

- ➢ PQ is a straight line. Not drawn accurately.
  Calculate the size of angle x. Do not use a protractor (angle measurer).

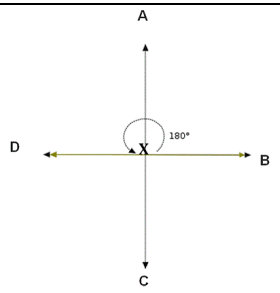


- ➢ This shape is three-quarters of a circle.

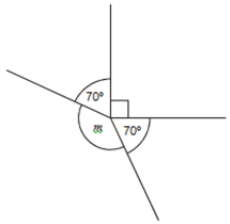


How many degrees is angle x?

- ➢ In the diagram below, if you were standing at X, facing A, what angle would you turn through if you turn and face C?

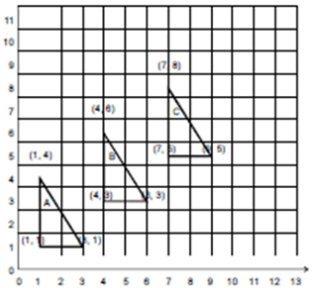


- This diagram is not drawn accurately. Calculate the size of angle m

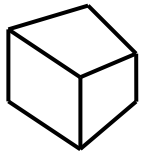
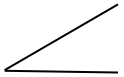


Use the properties of rectangles to deduce related facts and find missing lengths and angles
Distinguish between regular and irregular polygons based on reasoning about equal sides and angles

identify, describe and represent the position of a shape following a reflection or translation, using the appropriate language, and know that the shape has not changed



Write the co-ordinates of the next triangle in the sequence.

Reasoning opportunities and probing questions	Mathematical Language	Possible misconceptions
• (Showing photograph / sketch / isometric drawing / net), convince me that this shape is a cuboid / cube / prism / … • Show me a way to draw a cube. And another. And another … • Show me a way to draw a 2cm by 3cm by 4cm cuboid on isometric paper. And another. And another … • What is wrong with this sketch of a cuboid? How can it be changed?  • NCETM: Geometry - Properties of Shapes Reasoning • Convince me that a square is a rectangle • Show me an example of a hexagon. And another, and another, … • What is the same and what is different:  • Show me an acute (obtuse, reflex) angle. And another. And another. • Jenny uses a protractor to measure this angle:  She writes down 140°. Do you agree with Jenny? • Convince me how to measure a reflex angle using a 180° protractor. • Kenny thinks that 90° is an acute angle. Jenny thinks that 90° is an obtuse angle. Who is correct? Explain your answer. • (Given a grid with the point (6, 1) indicated) Benny describes this point as (1, 6). Jenny describes the point as (6, 1). Who do you agree with? Why? • Two vertices of a rectangle are (5, 2) and (4, 0). What could the other two vertices be? How many solutions can you find? • Always / Sometimes / Never: A mirror line touches the shape that is being reflected • Always / Sometimes / Never: Translations are easier than reflections	Cube Cuboid Cylinder Pyramid Prism Cone Sphere 2D 3D Net Sketch Isometric paper Rectangle Square Quadrilateral (Regular / irregular) polygon, pentagon, hexagon, octagon (Right) angle Parallel Perpendicular Coordinates Turn Angle Degrees Right angle Acute angle Obtuse angle Reflex angle Protractor Grid Axis, axes, x-axis, y-axis Origin (First) quadrant (Cartesian) coordinates Point Translation Reflection Transformation Object, Image Congruent, congruence	• Pupils must have isometric paper in portrait orientation for it to work correctly. • When drawing a cube on isometric paper, some students may think that they need to join dots to make a square first, and will draw horizontal and vertical lines to attempt to achieve this • Correct use of isometric paper must not indicate 'hidden' lines • Some pupils may think that a 'regular' polygon is a 'normal' polygon • Some pupils may think that all polygons have to be regular • Some pupils may use coordinates the wrong way round; for example, interpreting the point (3,2) as 3 up and 2 across (to the right) • Some pupils use the wrong scale on a protractor. For example, they measure an obtuse angle as 60° rather than 120°. • Some pupils may think that 90° is either an acute or obtuse angle. • Some pupils may think it is not possible to measure a reflex angle. • When describing or carrying out a translation, some pupils may count the squares between the two shapes rather than the squares that describe the movement between the two shapes. • When carrying out a reflection some pupils may think that the object and image should be an equal distance from the edge of the grid, rather than an equal distance form the mirror line. • Some pupils will confuse the order of x-coordinates and y-coordinates • When constructing axes, some pupils may not realise the importance of equal divisions on the axes

Statistics

Key concepts (National Curriculum statements)
Pupils should be taught to:

- solve comparison, sum and difference problems using information presented in a line graph
- complete, read and interpret information in tables, including timetables.

Notes and guidance (non-statutory)

- Pupils connect their work on coordinates and scales to their interpretation of time graphs.
- They begin to decide which representations of data are most appropriate and why.

Possible Themes

- Solve problems involving graphs

Key learning points

- Understand the difference between a line graph and a bar-line chart
- Identify when a line graph is an appropriate way to show data
- Read values from a line graph
- Answer one-step questions about data in line graphs (e.g. 'How much?')
- Answer two-step questions about data in line graphs (e.g. 'How much more?')
- Solve problems using information presented in a line graph
- Solve problems involving graphs

Prerequisite

✓ Interpret and construct a simple bar chart

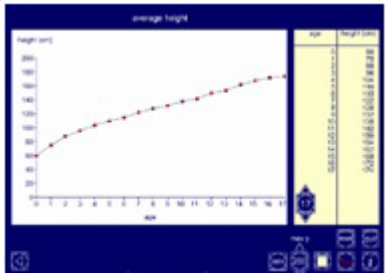
NCETM – Ready to Progress

complete, read and interpret information in tables, including timetables

- ➢ I can find the information in a table or graph to answer a question
- ➢ The table shows the cost of coach tickets to different cities.
- ➢ What is the total cost for a return journey to York for one adult and two children?

		Hull	York	Leeds
Adult	single	£12.50	£15.60	£10.25
	return	£23.75	£28.50	£19.30
Child	single	£8.50	£10.80	£8.25
	return	£14.90	£17.90	£14.75

Solve comparison, sum and difference problems using information presented in a line graph.



- ➢ What is the average height of children of different ages?
- ➢ Are there differences for boys and girls?

This screen shot is from the Interactive Teaching Programme 'Data Handling', using the 'Average Height' data set.

Reasoning opportunities and probing questions	Mathematical Language	Possible misconceptions
• Show me a line graph and tell me a story about it. And another. And another. • What is the same and what is different: Bar chart, bar-line chart, time graph, line graph? • Convince me that a line graph is not the same as a bar-line graph. • NCETM: Statistics Reasoning	Data Scale Axis Graph Frequency Time graph, Time series Line graph Bar-line graph, vertical line chart Maximum, minimum	• Some pupils may think that a line graph is appropriate for discrete data • Some pupils may think that a line graph is the same a bar-line chart • Some pupils may think that one centimetre represents one unit.