# Primary Mathematics Scheme of Work: Class 5
## Year 6

| | Week 1 | Week 2 | Week 3 | Week 4 | Week 5 | Week 6 | Week 7 |
|---|---|---|---|---|---|---|---|
| Autumn 1 | Number: Place Value | | Number: Addition, Subtraction, Multiplication and Division | | | | |
| Autumn 2 | Number: Fractions | | | Measurement: Converting Units of measure | | Number: Fractions | |
| Spring 1 | Number: Decimals | | Number: Fractions, Decimals and Percentages | | | Number: Ratio | |
| Spring 2 | Measurement: Area, Perimeter and Volume | | Statistics | Number: Algebra | Geometry: Shape | Geometry: Position and Direction | |
| Summer 1 | Number: Consolidation | | Geometry: Shape | | Geometry: Position and Direction | | |
| Summer 2 | Number: Fractions, Decimals and Percentages | | Number: Ratio | | Number: Algebra | | |

# Year 6

## *Place Value*

**Key concepts (National Curriculum statements)**

- identify the value of each digit in numbers given to three decimal places and multiply and divide numbers by 10, 100 and 1000 giving answers up to three decimal places
- read, write, order and compare numbers up to 10 000 000 and determine the value of each digit
- use negative numbers in context, and calculate intervals across zero
- identify common factors, common multiples and prime numbers

**Notes and guidance (non-statutory)**

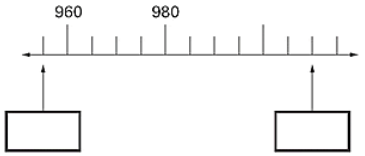
- Pupils use the whole number system, including saying, reading and writing numbers accurately.
- Common factors can be related to finding equivalent fractions.

| Possible Themes | Key learning points |
|---|---|
| • Understand and use decimals with up to three decimal places<br>• Work with numbers up to ten million<br>• Explore the use of negative numbers<br>• Develop understanding of factors and multiples<br>• Investigate prime numbers | • Write and read numbers up to and including 10 000 000<br>• Compare and order numbers up to and including 10 000 000<br>• Multiply numbers by 10<br>• Multiply numbers by 100<br>• Multiply numbers by 1000<br>• Divide numbers by 10<br>• Divide numbers by 100<br>• Divide numbers by 1000<br>• Understand and use negative numbers when working in context, such as temperature<br>• Calculate intervals across zero<br>• Find common multiples of two numbers<br>• Find common factors of two numbers<br>• Approximate any number by rounding to a specified degree of accuracy; e.g. nearest 1, 10, 100, 1000, decimal place, etc<br>• Understand estimating as the process of finding a rough value of an answer or calculation<br>• Use estimation to predict the order of magnitude of the solution to a decimal calculation, including decimals<br>• Check the order of magnitude of the solution to a calculation, including decimals |
| Prerequisite | NCETM – Ready to Progress |

- ✓ Understand and use place value in numbers with up to seven digits
- ✓ Multiply and divide whole numbers by 10, 100, 1000
- ✓ Multiply and divide numbers with one decimal place by 10, 100, 1000
- ✓ Know the meaning of 'factor' and 'multiple' and 'prime'

Read, write, order and compare numbers up to 10 000 000 and determine the value of each digit

- Children should be able to determine the steps used in different scales, and so complete activities such as;




Round any whole number to a required degree of accuracy

- Children should be able to circle the best estimate of the answer to questions such as;

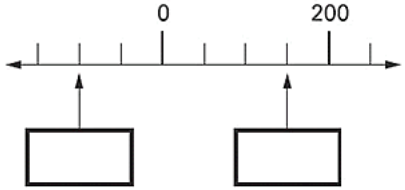
  72.34 ÷ 8.91

  When given

  6 7 8 9 10 11 as possible answers
- Children should estimate the position of numbers on a number line. They should suggest which number lies about two-fifths of the way along a line from 0 to 1000, or a line from 0 to 1. They should be able to justify their decisions.

Use negative numbers in context, and calculate intervals across zero

- Children should be able to work with negative numbers in a similar way, determining values on a scale and estimating.




Solve number and practical problems that involve all of the above

- Children should be able to use rounding and inverse operations to estimate and check calculations such as;
- The temperature inside an aeroplane is 20°C The temperatures outside the aeroplane is -30°C. What is the difference between these temperatures?

Solve problems involving multiplication and division

Use estimation to check answers to calculations and determine, in the context of a problem, levels of accuracy

Children should be able to:

- Give the best approximation to work out 4.4 × 18.6 and explain why. Answer questions such as: roughly, what answer do you expect to get? How did you arrive at that estimate? Do you expect your answer to be greater or less than your estimate? Why?

| Reasoning opportunities and probing questions | Mathematical Language | Possible misconceptions |
|---|---|---|
| • Convince me that 109 is a prime number<br>• Jenny writes 2.54 × 10 = 25.4. Kenny writes 2.54 × 10 = 25.40. who do you agree with? Explain why.<br>• Look at this number (24 054 028). Show me another number (with 4, 5, 6, 7 digits) that includes a 5 with the same value. And another. And another …<br>• Convince me a common factor of 12 and 30 is 6. | Place value<br>Digit<br>Negative number<br>(Common) multiple<br>(Common) factor<br>Divisible<br>Prime number, Composite number | • Some pupils confuse factors and multiples.<br>• Some pupils can confuse the language of large (and small) numbers since the prefix 'milli- means 'one thousandth' (meaning that there are 1000 millimetres in a metre for example) while one million is actually a thousand thousand.<br>• Some pupils may not realise that degrees (°) and degrees Celsius (°C) are two different and distinct units of measurement<br>• Some pupils may think that 1 is a prime number |

## *Addition, Subtraction, Multiplication and Division*

**Key concepts (National Curriculum statements)**
**Pupils should be taught to:**

- multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication
- divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context
- divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context
- perform mental calculations, including with mixed operations and large numbers
- identify common factors, common multiples and prime numbers
- use their knowledge of the order of operations to carry out calculations involving the four operations
- solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why

**Notes and guidance (non-statutory)**

- Pupils practise addition, subtraction, multiplication and division for larger numbers, using the formal written methods of columnar addition and subtraction, short and long multiplication, and short and long division
- They undertake mental calculations with increasingly large numbers and more complex calculations.
- Pupils continue to use all the multiplication tables to calculate mathematical statements in order to maintain their fluency.
- Pupils round answers to a specified degree of accuracy, for example, to the nearest 10, 20, 50 etc., but not to a specified number of significant figures.
- Pupils explore the order of operations using brackets; for example, 2 + 1 x 3 = 5 and (2 + 1) x 3 = 9.
- Common factors can be related to finding equivalent fractions.

| Possible Themes | Key learning points |
|---|---|
| • Develop mental calculation skills<br>• Extend written methods of multiplication<br>• Know and use the order of operations<br>• Solve problems involving addition, subtraction and multiplication<br>• Develop written methods of short division for numbers up to four-digits divided by a one-digit number<br>• Deal with remainders when carrying out division<br>• Solve problems involving the four operations | • Carry out calculations mentally involving numbers up to 4 digits.<br>• Multiply a four-digit number by a two-digit number using long multiplication<br>• Carry out calculations involving a mixture of multiplication and division<br>• Carry out calculations involving mixture of addition and subtraction<br>• Carry out calculations involving mixture of multiplication and addition/subtraction<br>• Carry out calculations involving mixture of division and addition/subtraction<br>• Solve multi-step problems involving addition, subtraction and/or multiplication<br>• Divide a three-digit number by a two-digit number using a formal written method of division with no remainder<br>• Divide a three-digit number by a two-digit number using a formal written method of division with a remainder<br>• Divide a four-digit number by a two-digit number using a formal written method of division with no remainder<br>• Divide a four-digit number by a two-digit number using a formal written method of division with a remainder<br>• Understand how to write the remainder to a division problem as a whole number remainder or as a fraction<br>• Understand how to interpret remainder to a division problem appropriately for the context<br>• Solve problems involving division |
| Prerequisite | NCETM – Ready to Progress |

- ✓ Recall multiplication facts for multiplication tables up to 12 × 12
- ✓ Recall division facts for multiplication tables up to 12 × 12
- ✓ Understand the commutativity of multiplication and addition
- ✓ Multiply a three-digit number by a two-digit number using long multiplication
- ✓ Use column addition and subtraction for numbers with more than four digits
- ✓ Use knowledge of multiplication tables when dividing
- ✓ Know how to use short division

solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why

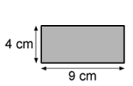
- Two numbers have a difference of 1.583. One of the numbers is 4.728. What is the other? Is this the only answer?

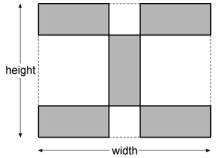
solve problems involving addition, subtraction, multiplication and division

- use estimation to check answers to calculations and determine, in the context of a problem, levels of accuracy
- Identify subtractions they can do without writing anything down

Identify why it is possible to solve a calculation mentally, explain the clues they looked for and then solve it

- Peter has £10. He buys 3 kg of potatoes at 87p per kg and 750 g of tomatoes at £1.32 per kg. How much money does he have left?
- Each tile is 4 centimetres by 9 centimetres.




- Calculate the width and height of the design.
- Write down the calculations that you did

multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication

- Look at long-multiplication calculations containing errors, identify the errors and determine how they should be corrected.
- Solve problems such as:Printing charges for a book are 3p per page and 75p for the cover. I paid £4.35 to get this book printed. How many pages are there in the book? Write down the calculations that you did. Seeds are £1.45 for a packet. I have £10 to spend on seeds. What is the greatest number of packets I can buy?

perform mental calculations, including with mixed operations and large numbers

- Use mental strategies to calculate in their heads, using jottings and/or diagrams where appropriate. For example, to calculate 24 × 15, they multiply 24 × 10 and then halve this to get 24 × 5, adding these two results together. They record their method as (24 × 10) + (24 × 5). Alternatively, they work out 24 × 5 = 120 (half of 24 × 10), then multiply 120 by 3 to get 360.
- identify common factors, common multiples and prime numbers
- Children should be able to answer questions such as:
- How can you use factors to multiply 17 by 12?
- Start from a two-digit number with at least six factors, e.g. 72. How many different multiplication and division facts can you make using what you know about 72? What facts involving decimals can you derive?
- What if you started with 7.2? What about 0.72?
- Which three prime numbers multiply to make 231?

use their knowledge of the order of operations to carry out calculations involving the four operations

- Children should be able to find answers to calculations such as 5.6 ☐ = 0.7 or 3 × 0.6, drawing on their knowledge of number facts and understanding of place value. They should be able to approximate, use inverses and apply tests of divisibility to check their results.
- Children should know the square numbers up to 12 × 12 and derive the corresponding squares of multiples of 10, for example 80 × 80 = 6400.

divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context

- Every day a machine makes 100 000 paper clips, which go into boxes. A full box has 120 paper clips. How many full boxes can be made from 100 000 paper clips?
- Each paper clip is made from 9.2 centimetres of wire. What is the greatest number of paper clips that can be made from 10 metres of wire?
- A DJ has two different sized storage boxes for her CDs. Small boxes hold 15 CDs. Large boxes hold 28 CDs. The DJ has 411 CDs. How could the DJ pack her CDs?

| Reasoning opportunities and probing questions | Mathematical Language | Possible misconceptions |
|---|---|---|

| | | |
|---|---|---|
| • Find missing digits in otherwise completed long multiplication calculations<br>• Convince me that 2472 × 12 = 29664<br>• Why have you chosen to add (subtract, multiply)?<br><br>NCETM: Addition and Subtraction Reasoning<br>• NCETM: Multiplication and Division Reasoning<br>• Find missing digits in otherwise completed long / short division calculations<br>• Show me a calculation that is connected to 147 × 26 = 3822. And another, and another …<br>• Show me a division calculation that has no remainder. Now show me a division by a two-digit number that has no remainder. And now, a four-digit number divided by a two-digit number that has no remainder. And now, with a remainder of 3 … | Addition<br>Subtraction<br>Sum, Total<br>Difference, Minus, Less<br>Column addition<br>Column subtraction<br>Operation<br>Multiply, Multiplication<br>Times<br>Product<br>Commutative<br>Factor<br>Short multiplication<br>Long multiplication<br>Estimate<br>Commutative<br>Divide, Division, Divisible<br>Divisor, Dividend, Quotient, Remainder<br>Factor<br>Short division<br>Long division<br>Remainder<br>Operation<br>Estimate | • Some pupils may write statements such as 140 - 190 = 50<br>• When subtracting mentally some pupils may deal with columns separately and not combine correctly; e.g. 180 – 24: 180 – 20 = 160. Taking away 4 will leave 6. So the answer is 166.<br>• The use of BIDMAS (or BODMAS) can imply that division takes priority over multiplication, and that addition takes priority over subtraction. This can result in incorrect calculations.<br>• Some pupils may write statements such as 12 ÷ 132 = 11<br>• Formal written methods of addition, subtraction and multiplication work from right to left. Formal division works from left to right.<br>• When using short division many pupils will at first struggle to deal correctly with any division where the divisor is greater than the first digit of the dividend; for example:<br>0 10 7 r 5<br>8 \| 3 $^{8}$6 $^{6}$1<br>• 3 ÷ 8 = 0 remainder 3, and so the 3 should be moved across. Instead, the 8 has been 'moved across' and therefore everything that follows has been correctly carried out based on an early misunderstanding. |

## *Fractions, Decimals and Percentages*

**Key concepts (National Curriculum statements)**
**Pupils should be taught to:**

- use common factors to simplify fractions; use common multiples to express fractions in the same denomination
- compare and order fractions, including fractions > 1
- add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions
- multiply simple pairs of proper fractions, writing the answer in its simplest form
- divide proper fractions by whole numbers
- associate a fraction with division and calculate decimal fraction equivalents
- identify the value of each digit in numbers given to three decimal places and multiply and divide numbers by 10, 100 and 1000 giving answers up to three decimal places

**Notes and guidance (non-statutory)**

- multiply one-digit numbers with up to two decimal places by whole numbers
- use written division methods in cases where the answer has up to two decimal places
- solve problems which require answers to be rounded to specified degrees of accuracy
- recall and use equivalences between simple fractions, decimals and percentages, including in different contexts.

| Possible Themes | Key learning points |
|---|---|

| | |
|---|---|
| • Explore the equivalence between fractions<br>• Use the equivalence between fractions<br>• Explore the equivalence between fractions, decimals and percentages<br><br>• Calculate with fractions<br>• Calculate with decimals<br>• Calculate with percentages | • Use common factors to simplify fractions<br>• Use common multiples to find equivalent fractions<br>• Compare and order fractions<br>• Compare and order fractions, including fractions > 1<br>• Understand a fraction is associated with division<br>• Work out the decimal equivalents of fifths, eighths and tenths<br>• Know simple fractions, decimals and percentages equivalences (e.g. 10%, 20%, 25%, 50%, 75%, 100%)<br>• Find equivalencies between fractions, decimals and percentages<br>• Add fractions with different denominators<br>• Add a mixed number and a fraction, including with different denominators<br>• Add mixed numbers, including with different denominators<br>• Subtract fractions with different denominators<br>• Subtract a mixed number and a fraction, including with different denominators<br>• Subtract mixed numbers, including with different denominators<br>• Multiply a proper fraction by a proper fraction<br>• Divide a proper fraction by a whole number<br>• Multiply U.t by U<br>• Multiply U.th by U<br>• Calculate percentages of a quantity<br>• Solve problems involving the use of percentages to make comparisons |
| Prerequisite | NCETM – Ready to Progress |

- ✓ Understand the concept of a fraction as a proportion
- ✓ Understand the concept of equivalent fractions
- ✓ Understand the concept of fractions, decimals and percentages being equivalent
- ✓ Know standard fraction / decimal equivalences (e.g. ½ = 0.5, ¼ = 0.25, $^{1}/_{10}$ = 0.1)
- ✓ Know that a percentage means 'out of 100'
- ✓ Convert between mixed numbers and improper fractions
- ✓ Find equivalent fractions
- ✓ Add and subtract fractions when one denominator is a multiple of the other
- ✓ Multiply a proper fraction by a whole number
- ✓ Use the formal written method of short multiplication
- ✓ Know the effect of multiplying and dividing by 10 and 100
- ✓ Know percentage equivalents of $^{1}/_{2}$, $^{1}/_{4}$, $^{3}/_{4}$, $^{1}/_{5}$, $^{2}/_{5}$, $^{4}/_{5}$

use common factors to simplify fractions; use common multiples to express fractions in the same denomination

- Children should be able to recognise that a fraction such as $^{5}/_{20}$ can be reduced to an equivalent fraction of ¼ by dividing both numerator and denominator by the same number [cancelling] They should also be familiar with identifying fractions in different units. E.g. what fraction is 20 pence of two pounds? Of four pounds etc…

compare and order fractions, including fractions >1

Children should be able to:

- i] Position fractions on a number line; e.g. mark fractions such as $^{7}/_{5}$ , $^{11}/_{20}$ , $^{18}/_{12}$ on a number line graduated in tenths
- ii] Answer questions such as: What number is half way between 5 ¼ and 5 ½ ?
- iii] Which is larger, ⅓ or ⅖? Explain how you know.

associate a fraction with division to calculate decimal fraction equivalents (e.g. 0.375) for a simple fraction (e.g. ⅜)

- Children should be able to find fractions of numbers and quantities;
  i] What fraction of £1 is 35p, … 170p ?
  ii] Write $^{23}/_{100}$ of 4 kilogrammes in grams
  iii] What fraction of 1 litre is 413 ml?
- Convert a fraction to a decimal using known equivalent fractions:
  i] ¼ = 0.25
  ii] ⅖ = 0.4

associate a fraction with division and calculate decimal fraction equivalents (e.g. 0.375) for a simple fraction (e.g. $^{3}/_{8}$)

Children should be able to:

- Explain how much pizza each person would get if they divided 4 pizzas between 5 people, as a fraction and a decimal

recall and use equivalences between simple fractions, decimals and percentages, including in different contexts.

- Children should be able to put a ring around the percentage that is equal to three-fifths;
- 20% 30% 40% 50% 60%
- As well as circle the two fractions that are equivalent to 0.6.
  $^{6}/_{10}$ $^{1}/_{60}$ $^{60}/_{100}$ $^{1}/_{6}$

add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions

- Children should be able to solve practical problems such as;

- Here is a chocolate bar.
- William eats 3 pieces and Amber eats 2 pieces. What fraction of the chocolate bar remains?
- Joe has some pocket money. He spends three-quarters of it. He has fifty pence left. How much pocket money did he have?

multiply simple pairs of proper fractions, writing the answer in its simplest form, (e.g. ¼ × ½ = ⅛)

- Children should be able to:
  ] Recognise that ¼ of 12, ¼ x 12 and 12 divided by 4 are equivalent
  ii] Use cancellation to simplify the product of a fraction and an integer
  eg ⅕ x 15 = 3
  ⅖ x 15 = 2 x ⅕ x 15 = 2x3 = 6
  ii] Work out how many ⅕s in 15, how many ⅖s in 15, how many 2/5s in 1 etc.

divide proper fractions by whole numbers (e.g. ⅓ ÷ 2 = ⅙ )

Children should be able to:

- Decide whether they would prefer to share ½ of a pizza with 2 people or ¾ of a pizza with 4 people and explain why.

associate a fraction with division and calculate decimal fraction equivalents (e.g. 0.375) for a simple fraction (e.g. $^{3}/_{8}$)

Children should be able to:

- Explain how much pizza each person would get if they divided 4 pizzas between 5 people, as a fraction and a decimal

identify the value of each digit to three decimal places and multiply and divide numbers by 10, 100 and 1000 where the answers are up to three decimal places

- Children should be able to identify the value of each digit in the number 17.036 and multiply and divide this by 10.100 and 1000

multiply one-digit numbers with up to two decimal places by whole numbers

- Children should be able to calculate the answer to questions such as;

| | |
|---|---|
| | ➢ What is 3.86 multiplied by nine?<br>use written division methods in cases where the answer has up to two decimal places<br>➢ Children should be able to calculate 601 divided by 36, to two decimal places<br>solve problems involving the calculation of percentages (e.g. of measures) such as 15% of 360 and the use of percentages for comparison<br>➢ Find simple percentages of amounts and compare them. For example:<br>➢ A class contains 12 boys and 18 girls. What percentage of the class are girls? What percentage are boys?<br>25% of the apples in a basket are red. The rest are green. There are 21 red apples. How many green apples are there? |

| Reasoning opportunities and probing questions | Mathematical Language | Possible misconceptions |
|---|---|---|
| • Show me another fraction that is equivalent to this one. And another. And another …<br>• Convince me that 3/8 = 0.375<br>• If you know that 1/10 = 0.1 = 10%, what else can you work out?<br>• Jenny is simplifying fractions. She has the fraction 16/64. Jenny says, 'if I cancel out the sixes then 16/64 = 1/4.'. Do you agree with Jenny? Why?<br>• NCETM: Fractions Reasoning<br>• Show me an 'easy' ('difficult') pair of fractions to add (subtract). And another. And another.<br>• Kenny thinks that $\frac{7}{10} - \frac{2}{7} = \frac{5}{3} = 1\frac{2}{3}$. Do you agree with Kenny?<br>• Jenny thinks that you can only multiply fractions if they have the same common denominator. Do you agree with Jenny? Explain.<br>• Benny thinks that $\frac{4}{10} \div 2 = \frac{4}{5}$. Do you agree with Kenny? Explain.<br>• Lenny says *'20% of £60 is £3 because 60 ÷20 = 3'*. Do you agree?<br>• | Fraction<br>Improper fraction, Proper fraction, Vulgar fraction, Top-heavy fraction<br>Percentage<br>Decimal<br>Proportion<br>Simplify<br>Equivalent<br>Lowest terms | • A fraction can be visualised as divisions of a shape (especially a circle) but some pupils may not recognise that these divisions must be equal in size, or that they can be divisions of any shape.<br>• Pupils may not make the connection that a percentage is a different way of describing a proportion<br>• Some pupils may think that simplifying a fraction just requires searching for, and removing, a factor of 2 (repeatedly)<br>• Some pupils may think that you simply can simply add/subtract the whole number part of mixed numbers and add/subtract the fractional art of mixed numbers when adding/subtracting mixed numbers, e.g. $3\frac{1}{3} - 2\frac{1}{2} = 1\frac{-1}{6}$<br>• Some pupils may make multiplying fractions over complicated by applying the same process for adding and subtracting of finding common denominators.<br>• Some pupils may think that as you divide by 10 to find 10%, you divide by 15 to find 15%, divide by 20 to find 20%, divide by 100 to find 100%, etc. |

## *Ratio and Proportion*

**Key concepts (National Curriculum statements)**
**Pupils should be taught to:**

- solve problems involving the relative sizes of two quantities where missing values can be found by using integer multiplication and division facts
- solve problems involving the calculation of percentages [for example, of measures and such as 15% of 360] and the use of percentages for comparison
- solve problems involving similar shapes where the scale factor is known or can be found
- solve problems involving unequal sharing and grouping using knowledge of fractions and multiples.

**Notes and guidance (non-statutory)**

- Pupils recognise proportionality in contexts when the relations between quantities are in the same ratio (for example, similar shapes and recipes).
- Pupils link percentages or 360° to calculating angles of pie charts.
- Pupils should consolidate their understanding of ratio when comparing quantities, sizes and scale drawings by solving a variety of problems. They might use the notation a:b to record their work.
- Pupils solve problems involving unequal quantities, for example, 'for every egg you need three spoonfuls of flour', ' 5 3 of the class are boys'. These problems are the foundation for later formal approaches to ratio and proportion.

| Possible Themes | Key learning points |
|---|---|

- Solve problems involving scaling
- Explore enlargement
- Solve problems involving sharing and grouping

- Solve simple problems involving the relative sizes of two quantities where missing values can be found by using integer multiplication and division facts; e.g. find the value of the parts, given the whole)
- Solve problems involving the relative sizes of two quantities where missing values can be found by using integer multiplication and division facts; e.g. find the value of the whole and parts, given one part)
- Use a scale factor to solve problems involving similar shapes
- Find the scale factor of similar shapes
- Solve problems involving unequal sharing or grouping problems using fractions
- Solve problems involving unequal sharing or grouping problems using multiples

## Prerequisite

- ✓ Recall multiplication facts for multiplication tables up to 12 × 12
- ✓ Recall division facts for multiplication tables up to 12 × 12
- ✓ Find fractions of an amount
- ✓ Find multiples of a given number

## NCETM – Ready to Progress

solve problems involving the relative sizes of two quantities where missing values can be found by using integer multiplication and division facts

Answer problems such as:

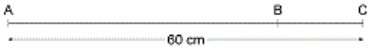
- Here is a recipe for pasta sauce.

  **Pasta sauce**
  300 g tomatoes
  120 g onions
  75 g mushrooms
  Sam makes the pasta sauce using 900 g of tomatoes. What weight of onions should he use? What weight of mushrooms?
- A recipe for 3 portions requires 150 g flour and 120 g sugar. Desi's solution to a problem says that for 2 portions he needs 80 g flour and 100 g sugar. What might Desi have done wrong? Work out the correct answer.
- This map has a scale of 1 cm to 6 km.


- The road from Ridlington to Carborough measured on the map is 6.6 cm long. What is the length of the road in kilometres?

solve problems involving similar shapes where the scale factor is known or can be found

- Solve simple problems involving direct proportion by scaling quantities up or down, for example:
- Two rulers cost 80 pence. How much do three rulers cost?
- Use the vocabulary of ratio and proportion to describe the relationships between two quantities solving problems such as:
- Two letters have a total weight of 120 grams. One letter weighs twice as much as the other. Write the weight of the heavier letter.
- The distance from A to B is three times as far as from B to C. The distance from A to C is 60 centimetres. Calculate the distance from A to B.




solve problems involving unequal sharing and grouping using knowledge of fractions and multiples

- Relate fractions to multiplication and division (e.g. 6 ÷ 2 = ½ of 6 = 6 × ½), simplify fractions by cancelling common factors, find fractions of whole-number quantities and solve problems such as:
- What fraction is 18 of 12
- What fraction is 500ml of 400ml?
- What is $^{14}/_{35}$ in its simplest form? ⅖
- What ⅓ × 15? What about 15 × ⅓? How did you work it out?
- What is two thirds of 66?

What is three quarters of 500?

## Reasoning opportunities and probing questions

## Mathematical Language

## Possible misconceptions

| | | |
|---|---|---|
| • (Given a recipe for 4 people) show me an amount of food that is needed for 8 people, 6 people, 9 people. Show me an amount of food that is needed for a number of people of your choice. And another. And another …<br>• Convince me that the second shape is an enlargement of the first shape<br>• Kenny has no sweets. Jenny gives 1/3 of her sweets to Kenny. Jenny now has 18 sweets. Kenny thinks that Jenny had 54 sweets to start with. Kenny is wrong. Explain why.<br><br>• NCETM: Ratio and Proportion Reasoning | Proportion<br>Quantity<br>Integer<br>Similar (shapes)<br>Enlargement<br>Scale factor<br>Group<br>Share<br>Multiples | • Many pupils will want to identify an additive relationship between two quantities that are in proportion and apply this to other quantities in order to find missing amounts<br>• When finding a fraction of an amount some pupils may try to use a rule formed without the necessary understanding. As a result they will muddle the operations, dividing by the numerator and multiplying by the denominator.<br>• When constructing an enlargement some pupils may only apply the scale factor in one dimension; for example, 'enlarging' a 2 by 4 rectangle by a scale factor of 2 and drawing a 2 by 8 rectangle. |

## *Algebra*

**Key concepts (National Curriculum statements)**
**Pupils should be taught to:**

- use simple formulae
- generate and describe linear number sequences
- express missing number problems algebraically
- find pairs of numbers that satisfy an equation with two unknowns
- enumerate possibilities of combinations of two variables.

**Notes and guidance (non-statutory)**

- missing numbers, lengths, coordinates and angles
- formulae in mathematics and science
- equivalent expressions (for example, a + b = b + a)
- generalisations of number patterns
- number puzzles (for example, what two numbers can add up to).

| Possible Themes | Key learning points |
|---|---|
| • Use simple formulae written in words<br>• Create simple formulae written in words<br>• Work with formulae written algebraically<br>• | • Use a simple one-step formula written in words<br>• Use a simple two-step formula written in words<br>• Use simple formula expressed in symbols<br>• Convert between miles and kilometres<br>• Recognise and describe a linear sequence<br>• Find the next terms in a linear sequence<br>• Find a missing term in a linear sequence<br>• Generate a linear sequence from its description |
| Prerequisite | NCETM – Ready to Progress |

| | |
|---|---|
| ✓ Know the order of operations<br>✓ Know the fact that area of rectangle = length × width | express missing number problems algebraically - use simple formulae<br>➢ Children should be able to express a relationship in symbols, and start to use simple formulae. For example:<br>➢ Use symbols to write a formula for the number of months m in y years.<br>➢ Write a formula for the cost of c chews at 4p each.<br>➢ Write a formula for the nth term of this sequence: 3, 6, 9, 12, 15…<br>➢ The perimeter of a rectangle is 2 × (l + b), where l is the length and b is the breadth of the rectangle.<br>➢ What is the perimeter if l = 8 cm and b = 5 cm?<br>➢ The number of bean sticks needed for a row which is m metres long is 2m + 1. How many bean sticks do you need for a row which is 60 metres long?<br>➢ Plot the points which show pairs of numbers with a sum of 9. |

| Reasoning opportunities and probing questions | Mathematical Language | Possible misconceptions |
|---|---|---|
| • Look at this formula. Write down a fact that it tells you. And another. And another …<br>• Jenny and Kenny are using the formula 'Cost in pounds = 40 + 20 × number of hours' to work out the cost for three hours. Jenny writes down £180. Kenny writes down £100. Who do you agree with? Why?<br>• Always / Sometimes / Never: The formula T = 4n + 6 results in an odd number.<br>• NCETM: Algebra Reasoning | Formula, Formulae<br>Expression<br>Variable<br>Substitute<br>Symbol<br>Mile<br>Kilometre<br>Metric<br>Imperial | • Some pupils may apply the order of operations incorrectly when working with two step formulae<br>• Units must be consistent when using formulae. For example, a mobile phone plan might charge £15 per month plus 5p for every text. The formula 'Monthly cost = 15 + 5 × number of texts is wrong because amounts in both pounds and pence are involved. Monthly cost (in pence) = 1500 + 5 × number of texts is one correct way of writing the formula.<br>• It is not advisable to abbreviate the formula 'kilometres = miles × 1.6' using letters. 'm' is the normal abbreviation for metres and 'k' can represent £1000. If 'km' is used it could even be interpreted as 'k × m'. |

## *Measurement*

**Key concepts (National Curriculum statements)**
**Pupils should be taught to:**

- solve problems involving the calculation and conversion of units of measure, using decimal notation up to three decimal places where appropriate
- use, read, write and convert between standard units, converting measurements of length, mass, volume and time from a smaller unit of measure to a larger unit, and vice versa, using decimal notation to up to three decimal places
- convert between miles and kilometres
- recognise that shapes with the same areas can have different perimeters and vice versa
- recognise when it is possible to use formulae for area and volume of shapes
- calculate the area of parallelograms and triangles
- calculate, estimate and compare volume of cubes and cuboids using standard units

**Notes and guidance (non-statutory)**

- Pupils connect conversion (for example, from kilometres to miles) to a graphical representation as preparation for understanding linear/proportional graphs.
- They know approximate conversions and are able to tell if an answer is sensible. Using the number line, pupils use, add and subtract positive and negative integers for measures such as temperature.

| Possible Themes | Key learning points |
|---|---|

| | |
|---|---|
| • Solve problems involving measurement<br>• Explore area<br>• Investigate volume<br>• Solve problems involving area and volume | • Convert between non-adjacent metric units length and mass from the smaller unit to the larger unit; e.g. centimetres to kilometres<br>• Convert between non-adjacent metric units length and mass from the larger unit to the smaller unit; e.g. kilometres and centimetres<br>• Convert between non-adjacent time units; e.g. hours to seconds<br>• Solve problems involving converting between measures<br>• Recognise that shapes with the same areas can have different perimeters and vice versa<br>• Calculate the area of a parallelogram<br>• Calculate the area of a triangle<br>• Estimate the volume of cubes and cuboids<br>• Calculate the volume of cuboid, including cubes<br>• Recognise when it is possible to use formulae to calculate area and volume<br>• Convert between metric units of area in simple cases<br>• Convert between metric units of volume in simple cases |
| Prerequisite | NCETM – Ready to Progress |

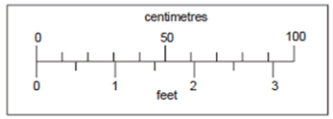
- ✓ Convert between adjacent metric units of length, mass and capacity
- ✓ Know rough equivalents between inches and cm, feet and cm, kg and lb, pint and ml
- ✓ Use decimal notation to two decimal places when converting between metric unit
- ✓ Know the meaning of perimeter (area, volume, capacity)
- ✓ Know that the area of a rectangle is given by the formula area = length × width
- ✓ Know that area can be measured using square centimetres or square metres, and the abbreviations $cm^2$ and $m^2$
- ✓ Know that volume is measured in cubes

solve problems involving the calculation and conversion of units of measure, using decimal notation up to three decimal places where appropriate

- ➢ Children should be able to draw a flow chart to help someone else convert between mm, cm, m and km.
- ➢ They should be able to answer questions such as: approximately how many litres are there in 3 gallons? Give your answer to the nearest litre.

use, read, write and convert between standard units, converting measurements of length, mass, volume and time from a smaller unit of measure to a larger unit, and vice versa, using decimal notation to up to three decimal places

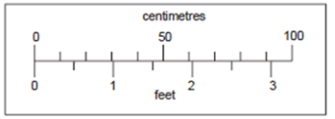
- ➢ This scale (not actual size) shows length measurements in centimetres and feet.
- ➢ Look at the scale. Estimate the number of centimetres that are equal to 2 ½ feet.
- ➢ Estimate the difference in centimetres between 50 cm and 1 foot.



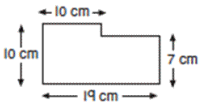

convert between miles and kilometres

- ➢ Pupils should know the approximate equivalence between commonly used imperial units and metric units:
  e.g. 1 litre is approximately 2 pints (more accurately, 1 ¾ pints)
  4.5 litres is approximately 1 gallon or 8 pints
  1 kilogram is approximately 2 lb (more accurately, 2.2 lb)
  30 grams is approximately 1 oz
  8 kilometres is approximately 5 miles
- ➢ Children should be able to use conversion graphs that show miles/kilometres. They should be able to use it to estimate a distance of 95 miles in kilometres.

- Children should be able to draw a flow chart to help someone else convert between mm, cm, m and km.
- They should be able to answer questions such as: approximately how many litres are there in 3 gallons? Give your answer to the nearest litre.
- This scale (not actual size) shows length measurements in centimetres and feet.
- Look at the scale. Estimate the number of centimetres that are equal to 2 ½ feet.
- Estimate the difference in centimetres between 50 cm and 1 foot.
- Pupils should know the approximate equivalence between commonly used imperial units and metric units:
  e.g. 1 litre is approximately 2 pints (more accurately, 1 ¾ pints), 4.5 litres is approximately 1 gallon or 8 pints, 1 kilogram is approximately 2 lb (more accurately, 2.2 lb), 30 grams is approximately 1 oz, 8 kilometres is approximately 5 miles




- Children should be able to use conversion graphs that show miles/kilometres. They should be able to use it to estimate a distance of 95 miles in kilometres.
- The perimeter of a square is 72 centimetres.
- The square is cut in half to make two identical rectangles.
- What is the perimeter of one rectangle?




- Children should be able to calculate the perimeters of compound shapes that can be split into rectangles. For example,

This is a centimetre grid. Draw 3 more lines to make a parallelogram with an area of $10cm^2$. Use a ruler.

| Reasoning opportunities and probing questions | Mathematical Language | Possible misconceptions |
|---|---|---|

| | | |
|---|---|---|
| • Show me a metric (imperial) unit of measure. And another. And another.<br>• Kenny thinks that 2.5km = 25 000 cm. Do you agree with Kenny? Explain your answer.<br>• Convince me that 4.25kg does not equal 425g.<br><br>• NCETM: Measurement Reasoning<br><br>• 'Show me' an example of when you would measure volume using $km^3$<br>• Convince me that the area of a parallelogram is found using base × height<br>• (Given a triangle with base labelled 8 cm, height 5 cm, slope height 6 cm) Kenny thinks that the area is 40 $cm^2$, Lenny thinks it is 20 $cm^2$, Jenny thinks it is 240 $cm^2$ and Benny thinks it is 24 $cm^2$. Who do you agree with? Explain why. | Length, distance<br>Mass, weight<br>Volume<br>Capacity<br>Metre, centimetre, millimetre<br>Tonne, kilogram, gram, milligram<br>Litre, millilitre<br>Hour, minute, second<br>Inch, foot, yard<br>Pound, ounce<br>Pint, gallon | • Some pupils may apply an incorrect understanding that there are 100 minutes in a hour when solving problems<br>• Some pupils may struggle when converting between 12- and 24-hour clock notation; e.g. thinking that 15:00 is 5 o' clock<br>• Some pupils may apply incorrect beliefs about place value, such as 2.3 × 10 = 2.30.<br>• Many conversions within the metric system rely on multiplying and dividing by 1000. The use of centimetres as an 'extra unit' within the system breaks this pattern. Consequently there is a frequent need to multiply and divide by 10 or 100, and this can cause confusion about the connections that need to be applied.<br>• Some pupils may use the sloping height when finding the areas of parallelograms and triangles<br>• Some pupils may think that the area of a triangle is found using area = base × height<br>• Some pupils may think that you multiply all the numbers to find the area of a shape |

## *Geometry*

**Key concepts (National Curriculum statements)**
**Pupils should be taught to:**

- draw 2-D shapes using given dimensions and angles
- recognise, describe and build simple 3-D shapes, including making nets
- compare and classify geometric shapes based on their properties and sizes and find unknown angles in any triangles, quadrilaterals, and regular polygons
- illustrate and name parts of circles, including radius, diameter and circumference and know that the diameter is twice the radius
- recognise angles where they meet at a point, are on a straight line, or are vertically opposite, and find missing angles.
- describe positions on the full coordinate grid (all four quadrants)
- draw and translate simple shapes on the coordinate plane, and reflect them in the axes.

**Notes and guidance (non-statutory)**

- Pupils draw shapes and nets accurately, using measuring tools and conventional markings and labels for lines and angles.
- Pupils describe the properties of shapes and explain how unknown angles and lengths can be derived from known measurements.
- These relationships might be expressed algebraically for example, d = 2 × r; a = 180 – (b + c).

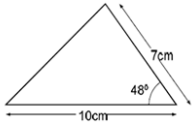
| Possible Themes | Key learning points |
|---|---|

| | |
|---|---|
| • Construct 2D shapes<br>• Investigate 3D shapes<br>• Explore nets of 3D shapes<br>• Develop knowledge of angles<br>• Apply angle facts to deduce unknown angles<br>• Understand and use Cartesian coordinates<br>• Use transformations to move shapes | • Draw 2-D shapes given angles<br>• Draw 2-D shapes given dimensions and angles<br>• Recognise prisms<br>• Recognise pyramids<br>• Classify 3-D shapes including cylinders, cones and spheres<br>• Draw nets of 3-D shapes<br>• Solve 3-D problems using nets including visualising the edges (vertices) that will meet when folded<br>• Classify 2D shapes using given categories; e.g. number of sides, symmetry<br>• Find unknown angles in a triangle<br>• Find unknown angles in an isosceles triangle when only one angle is known<br>• Find unknown angles in a quadrilateral<br>• Find unknown angles in regular polygons<br>• Solve problems involving 2-D shapes<br>• Know the names and relationships of the parts a circle<br>• Find missing angles where they meet at a point<br>• Find missing angles where they meet on a straight line<br>• Find missing angles where they are vertically opposite<br>• Use known facts to find missing angles<br>• Use coordinates to describe the position of a point in all four quadrants<br>• Use coordinates to plot the position of a point in any of the four quadrants<br>• Draw and translate simple shapes<br>• Carry out a reflection using one of the axes as a mirror line |
| Prerequisite | NCETM – Ready to Progress |

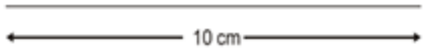
- ✓ Know the names of common 2D shapes
- ✓ Know the names of common 3D shapes
- ✓ Use a protractor to measure and draw angles
- ✓ Know the properties of rectangles
- ✓ Know the difference between a regular and an irregular polygon
- ✓ Add and subtract numbers up to three digits
- ✓ Know that angles are measured in degrees
- ✓ Know that angles in a full turn total 360°, and angle in half a turn must total 180°
- ✓ Estimate the size of angles
- ✓ Use coordinates in the first quadrant
- ✓ Identify a translation
- ✓ Carry out a translation in the first quadrant
- ✓ Identify a reflection
- ✓ Carry out a reflection in the first quadrant using mirror lines parallel to the axes
- ✓ Know the meaning of 'congruent', 'congruence', 'object', 'image'

draw 2-D shapes using given dimensions and angles

- Children should be able to construct a triangle given two sides and the included angle
- Here is a sketch of a triangle. (It is not drawn to scale).




Draw the full size triangle accurately, below. Use an angle measurer (protractor) and a ruler. One line has been drawn for you.




recognise, describe and build simple 3-D shapes, including making nets

- Children should be able to identify, visualise and describe properties of rectangles, triangles, regular polygons and 3-D solids; use knowledge of properties to draw 2-D shapes and identify and draw nets of 3-D shapes
- They should be able to respond accurately to questions such as;

'I am thinking of a 3D shape. It has a square base. It has four other faces which are triangles. What is the name of the 3D shape?'

'Which of these nets are of square based pyramids? How do you know?

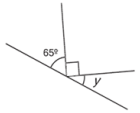
- 'Is this a net for an open cube?' How do you know?

compare and classify geometric shapes based on their properties and sizes and find unknown angles in any triangles, quadrilaterals, and regular polygons

Children should be able to make and draw shapes with increasing accuracy and knowledge of their properties.

- They should be able to carry out activities such as;
- 'Give me instructions to get me to draw a rhombus using my ruler and a protractor'
- 'On the grid below, use a ruler to draw a pentagon that has three right angles'
- Children should be able to calculate the size of angle 'y' in this diagram without using a protractor.

(Not to scale)




illustrate and name parts of circles, including radius, diameter and circumference and know that the diameter is twice the radius

They should know that:

- The circumference is the distance round the circle
- The radius is the distance from the centre to the circumference

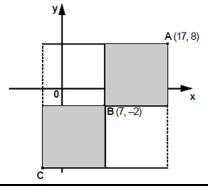
The diameter is 2 x radius

describe positions on the full coordinate grid (all four quadrants)

Children should be able to answer questions such as;

- The two shaded squares below are the same size.

A is the point (17,8). B is the point (7,-2).
What are the co-ordinates of the point C?



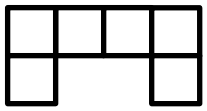
draw and translate simple shapes on the coordinate plane, and reflect them in the axes

- Children should be able to draw a shape with corners at given vertices, and describe the properties of the shape. Can they create the same shape where all of the coordinates will be positive? Negative?

They should be able to sketch the reflection of a simple shape in two mirror lines at right angles and find the coordinates of selected points.

## Reasoning opportunities and probing questions

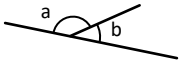
- Show me an example of a net of a cube. And another. And another …
- What is wrong with this attempt at a net of a cuboid? How can it be changed?



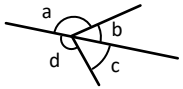
- How many different ways are there to complete these nets?
- Convince me a cylinder is not a prism.

- NCETM: Geometry - Properties of Shapes Reasoning

- Convince me that a rhombus is a parallelogram
- Jenny writes that 'Diameter = 2 × Radius'. Kenny writes that 'Radius = 2 × Diameter'. Who is correct?
- What is the same and what is different: a square and a rectangle?
- Show a pair of possible values for a and b.
  And another. And another.




- Convince me that the sum of angles on a straight line is 180°.
- Show a possible set of values for a, b, c and d.
  And another. And another.



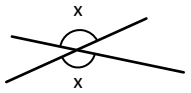

- Convince me that the sum of angles around a point is 360°.
- Convince me that (vertically) opposite angles are equal.
- Kenny thinks that the sum of opposite angles is 180°. Do you agree? Explain your answer.

## Mathematical Language

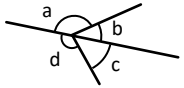
Protractor
Measure
Nearest
Construct
Sketch
Cube, Cuboid, Cylinder, Pyramid, Prism
Net
Edge, Face, Vertex (Vertices)
Visualise
Quadrilateral, Square, Rectangle, Parallelogram, (Isosceles) Trapezium, Kite, Rhombus, Delta, Arrowhead
Triangle, Scalene, Right-angled, Isosceles, Equilateral
Polygon, Regular, Irregular
Pentagon, Hexagon, Octagon, Decagon, Dodecagon
Circle, Radius, Diameter, Circumference, Centre
Parallel
Diagonal
Angle
2-D
Grid
Axis, axes, x-axis, y-axis
Origin
Quadrant
(Cartesian) coordinates
Point
Translation
Reflection
Transformation
Object, Image
Congruent, congruence

## Possible misconceptions

- Some pupils will read the wrong way round the scale on a typical semi-circular protractor, therefore using 180° - required angle
- Some pupils may measure from the end of a ruler, rather than the start of the measuring scale
- Some pupils may think that several repeats of a shape in any pattern constitutes a tessellation
- When given a net of a 3D shape some pupils may think that the number of vertices of the 3D shape is found by counting the number of 'corners' on the net
- Some pupils may think that a 'regular' polygon is a 'normal' polygon
- Some pupils may think that all polygons have to be regular
- Some pupils may think that a square is only square if 'horizontal', and even that a 'non-horizontal' square is called a diamond
- The equal angles of an isosceles triangle are not always the 'base angles' as some pupils may think
- Some pupils may think that these angles are not equal as they are not 'vertical'.




- Some pupils may think that angles that are 'roughly' opposite are always equal, e.g. a = c




- When describing or carrying out a translation, some pupils may count the squares between the two shapes rather than the squares that describe the movement between the two shapes.
- When reflecting a triangle some students may draw a translation
- When carrying out a reflection some pupils may think that the object and image should be an equal distance from the edge of the grid, rather than an equal distance form the mirror line.
- Some pupils will confuse the order of x-coordinates and y-coordinates

When constructing axes, some pupils may not realise the importance of equal divisions on the axes

## *Statistics*

**Key concepts (National Curriculum statements)**
**Pupils should be taught to:**

- interpret and construct pie charts and line graphs and use these to solve problems
- calculate and interpret the mean as an average.

**Notes and guidance (non-statutory)**

- Pupils connect their work on angles, fractions and percentages to the interpretation of pie charts.
- Pupils both encounter and draw graphs relating two variables, arising from their own enquiry and in other subjects.
- They should connect conversion from kilometres to miles in measurement to its graphical representation.
- Pupils know when it is appropriate to find the mean of a data set.

## Possible Themes

- Construct and interpret pie charts
- Solve problems involving graphs and charts

## Key learning points

- Interpret pie charts
- Construct a pie chart by measuring angles
- Interpret line graphs
- Construct line graphs

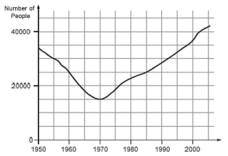
## Prerequisite

- ✓ Measure and construct angles using a protractor
- ✓ Interpret and construct a simple line graph

## NCETM – Ready to Progress

interpret and construct pie charts and line graphs and use these to solve problems

- This graph shows the number of people living in a town.




How many people lived in the town in 1985?
In which year was the number of people the same as in 1950?
Find the year when the number of people first went below 20 000. *KS2*
*2008 Paper A level 5*

- Class 6 did a survey of the number of trees in a country park. This pie chart shows their results.


Estimate the fraction of trees in the survey that are oak trees. The children counted 60 ash trees.
Use the pie chart to estimate the number of beech trees they counted. *KS2*
*2006 Paper A level 5*

## Reasoning opportunities and probing questions

- Show me a pie chart representing the following information: Blue (25%), Red (over 50%), Yellow (the rest). And another. And another.
- Always / Sometimes / Never: Pie charts are constructed in a clockwise direction
- Always / Sometimes / Never: The larger the size of the pie chart, the greater the total frequency
- Kenny says 'If two pie charts have the same section then the amount of data the section represents is the same in each pie chart.' Do you agree with Kenny? Explain your answer.
- NCETM: Statistics Reasoning

## Mathematical Language

Data
Scale
Axis, axes
Graph
Frequency
Time graph, Time series
Line graph
Pie chart
Sector
Angle
Protractor
Degrees
Maximum, minimum

## Possible misconceptions

- Some pupils may think the larger the size of the pie chart, the greater the total frequency
- Some pupils may think if two pie charts have the same section then the amount of data the section represents is the same in each pie chart.'
- Some pupils may confuse the fact that the sections of the pie chart total 100% and 360°
- Some pupils may think that a line graph is appropriate for discrete data
- Some pupils may think that each square on the grid used represents one unit